# Machine Scheduling Problem (MSP)

## 4. Exact Solution Methods

There are many methods of exact solution, in this thesis we will focus in complete enumeration, and branch and bound methods.

### 4.1 Complete Enumeration Method (CEM) [67]

Complete enumeration methods generate one by one, all feasible solutions and then pick the best one. For example, for a single machine problem of n jobs there are n! different sequences. Hence for the corresponding m machines problem, there are $(n!)^m$ different sequences. This method may take considerable time as the number $(n!)^m$ is very large even for relatively small values of n and m.

### 4.2 Branch and Bound Methods (BAB)

Branch and bound (BAB) methods are implicit enumeration techniques which can find an optimal solution by systematically examining subsets of feasible solutions. These methods are usually described by means of search tree with nodes that corresponding to these subsets. (BAB) methods were applied to scheduling problems by Ignall and Schrage [52] and Lomnicki [65].

The BAB method uses a search tree. Each node in the tree contains a partial sequence of jobs. For n job problem, there are n-1 numbers of levels for a tree. At level zero, root node will be placed with all n empty sequence positions. At level 1, there will be n number of nodes. Each node will contains a partial sequence of jobs. The first position in the sequence will be occupied by a job in numerical order. Similarly, each node at $(n-1)^{th}$ level will be branched to (n-2) number of nodes. The process will continue till each node has exactly one leaf [9].

Generation of all sequences is combinatorial in nature and, will result in enormous number of sequences even for a small number of jobs. For example, for a 10-job problem there will be 10! sequences. To reduce the computational effort, lower bounds are calculated at every level for each node. The formula used to compute the lower bound is pertained to objective function of the scheduling problem. Branching is carried out only from those nodes with a minimum lower bound. By doing so, only small proportion of the nodes is explored resulting in fewer amounts of computations. The BAB method is applied in almost every scheduling problem [9].

### 4.3 Dynamic Programing [4]

Dynamic programming (DP) method is implicit enumeration technique and it can be applied to any optimization problem which can be divided into a number of stages and at each stage the solution is derived by recurrence relation from the solution at the preceding stages.

There are some difficulties for this method, one of them is the difficulty of finding a good way for brake down problem into stages so that convenient computations are rather large, which means that the computation grows to exponential rate with increasing the size of problem [4].

## 5. Heuristic Methods for MSP

Reeves [78] defined the heuristic method as follows: **A heuristic** is a technique which seek good (i.e., near optimal) solution at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases to state how close to optimality a particular feasible solution (in section (2.5) we will give detail for the meta-heuristic or local search methods).

It is clear, to solve MSP one tends to use implicit enumerative approaches (BAB and dynamic programming) to find optimal solutions. However, these approaches have two disadvantages. Firstly, it is mathematically complex and thus a lot of time to be invested. Secondly, when it concerns an NP-hard problem, the computational requirements are enormous for large size problem, to avoid these drawbacks we can appeal to heuristic [78].

As most of scheduling problems are NP-hard, many researchers have developed heuristic algorithms to solve them in an efficient and effective way. In recent years, in practice, such a problem is often required to be solved in real time. Hence, a quick heuristic that allows a good feasible solution to be obtained in a predetermined and finite number of steps or in polynomial time is must desired. It is very hard to identify the borders between generic and special, feasible and infeasible, hard and easy, theoretical and practical. As a result, the real-life problems could be easily analyzed, modeled and solved by the right choice of standard tools from the toolbox under different scenarios [64].

The heuristic methods are applied at some steps of other methods (e.g. search tree method) to yield an upper bound (UB) or lower bound (LB) on the cost of an optimal schedule.

One of the important heuristic methods is the tree type heuristic method (TTHM). The branch and bound (BAB) method can be used to obtain upper bound on the optimal value of objective function, if some of the possible optimal partial schedules have not been explored. The TTHM

is using a BAB method without using backtracking procedure. The main step in this method the lower bound (LB) is evaluated at all nodes in each level of the search tree, then some of the nodes within each level of the search tree are chosen from which to branch. Usually, one node is chosen with each level and stop at the first complete sequence of the jobs to be the solution [82].