CHAPTER TWO

TRAVELING SALESMAN PROBLEM (TSP)

2.1 Traveling Salesman Problem (TSP) Concept

TSP Definition: Given a collection of cities and the cost of travel between each pair of them, the traveling salesman problem, or TSP for short, is to find the cheapest way of visiting all of the cities and returning to your starting point.

In the standard version we study, the travel costs are symmetric in the sense that traveling from city X to city Y costs just as much as traveling from Y to X. To put it differently, the data consist of integer weights assigned to the edges of a finite complete graph; the objective is to find a **Hamiltonian cycle** (that is, a cycle passing through all the vertices) of the minimum total weight. In this context, Hamiltonian cycles are commonly called **tours**.

The simplicity of the statement of the problem is deceptive - the TSP is one of the most intensely studied problems in computational mathematics and yet no effective solution method is known for the general case. Although the complexity of the TSP is still unknown, for over 50 years its study has led the way to improved solution methods in many areas of mathematical optimization. TSP is well-known NP-hard COP.

2.2 Applications of TSP

The TSP naturally arises as a subproblem in many transportation and logistics applications, for example:

- The problem of arranging school bus routes to pick up the children in a school district. This bus application is of important historical significance to the TSP, since it provided motivation for Merrill Flood, one of the pioneers of TSP research in the 1940s.

- A second TSP application from the 1940s involved the transportation of farming equipment from one location to another to test soil, leading to mathematical studies in Bengal by P. C. Mahalanobis and in Iowa by R. J. Jessen.
- More recent applications involve the scheduling of service calls at cable firms (Telephone routing and Networks).
- The delivery of meals to homebound persons.
- The scheduling of stacker cranes in warehouses.
- The routing of trucks for parcel post pickup, and a host of others.
- Manufacturing, plane routing and job sequencing.

2.3 Mathematical Formulation

The problem can be formulated in two prospects:

- Minimize travelling time.
- Minimize travelling distance.

The TSP can be represented as a network where the nodes and arcs represented the cities and the cities and the distance between them respectively.

Let there are n-cities and let d_{ij}, i,j=1,…,n (d_{ii}=0) be the distance between city i and city j. we defined the matrix X interpreted as:

$$x_{ij} = \begin{cases} 1, \text{if the route stars from node } i \text{ to } j \\ 0, \text{otherewise} \end{cases}$$

Then the Matrix X is:

$$X = \begin{bmatrix} 0 & x_{12} & \dots & x_{1n} \\ x_{21} & 0 & \dots & x_{2n} \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & 0 \end{bmatrix}$$

Then the mathematical formulation can be as follows:

$$\left.\begin{aligned} &\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} \\ &\text{Subject to:} \\ &\sum_{j=1}^{n} x_{ij} = 1, i = 1,\dots,n \\ &\sum_{i=1}^{n} x_{ij} = 1, j = 1,\dots,n \\ &x_{ij}=0,\ 1. \end{aligned}\right\} \qquad \dots(2.1)$$

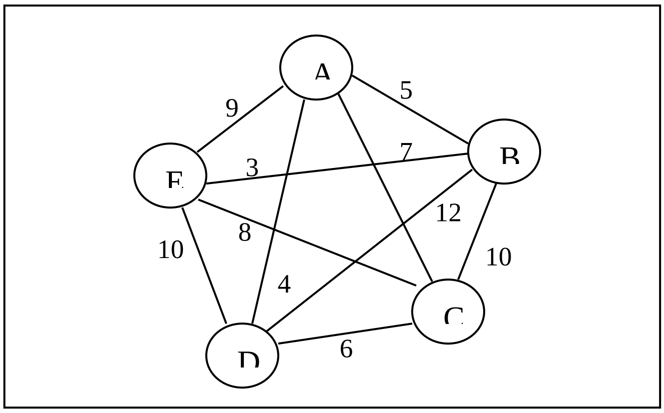
2.4 Representation of TSP

This problem can be represented in one of two ways (may both):

1. **Table form**: The distance between any the n-cities can be expressed in n×n matrix or table, for example let n=5:

	A	B	C	D	E
A	0	5	7	4	9
B	5	0	10	12	3
C	7	10	0	6	8
D	4	12	6	0	10
E	9	3	8	10	0

2. **Graph**: Every city can be expressed by node and the arc will represents the distance between cities, for the previous example,



Note that A-B is similar to B-A.

Here we want to find a rout (feasible solution) starting, for example, from A and end with A, e.g. the rout A-C-D-E-B-A. so the matrix X will be:

$$X = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Notice that each row and column has only one (1), s.t. the constraints of formula (2.1) are satisfied. When calculating Z by using formula (2.1) we obtain the cost:

$Z=d_{13}+d_{34}+ d_{45}+d_{52}+d_{21}=7+6+10+3+5=31$ units.