between chromosomes. For that purpose the crossover used in this algorithm is the **Order Crossover** (OX) [91], this operator chooses two random crossover points, for example, if the parents are:

$$\begin{array}{ll|lll|lll|lll} v_1 & : & 7\ 9\ 8 & 2\ 5\ 1 & 6\ 3\ 4 & \Rightarrow & 7\ 9\ * & 2\ 5\ 1 & 6\ *\ * \\ v_2 & : & 9\ 5\ 6 & 4\ 8\ 3 & 2\ 7\ 1 & & 9\ *\ 6 & 4\ 8\ 3 & *\ 7\ * \end{array}$$

Then put * instead of the digit repeated in the two sides of v_1 (or v_2) and in the middle of v_2 (or v_1), then distribute the sequence in the middle of v_1 (v_2) to the positions of * in the same v_1 (v_2).

$$\begin{array}{l|l|l|l|l|l|l} \Rightarrow & 7\ 9\ 2 & *\ *\ * & 5\ 1\ 6 & \Rightarrow\ o_1 & 7\ 9\ 2 & 4\ 8\ 3 & 5\ 1\ 6 \\ & 9\ 6\ 4 & *\ *\ * & 8\ 3\ 7 & \ o_2 & 9\ 6\ 4 & 2\ 5\ 1 & 8\ 3\ 7 \end{array}$$

Lastly, exchange the digits in middle of v_1 with v_2 to obtain o_1 and o_2.

- **Mutation Operator**

  After the new generation has been determined, the chromosomes are subjected to a low rate mutation process. For this example applies two mutation operators to introduce genetic diversity into the evolving population of permutation. The first operator is a simple two point mutation, which randomly selects two elements in the chromosome and swap them (1 10 **8** 4 5 6 7 9 **3** 2) becomes (1 10 **3** 4 5 6 7 9 **8** 2). The second operator is a shuffle mutation, which shunts the permutations forward by a random number of places; thus (**1 10 3 4 5** 6 7 9 8 2) shuffled forward six places becomes (6 7 9 3 2 **1 10 8 4 5**).

- **Genetic Parameters**

  For MSP, from our experience, the following parameters are preferred to be used: population size (pop_size=20), probability of crossover (Pc=0.7), probability of mutation Pm =0.1 and some hundreds number of generations.

P_1 1 2 | 3 4 | 5 ch_1 = 1 * | 3 4 | 5
P_2 5 4 | 3 2 | 1 ch_2 = 5 * | 3 2 | 1

ch_1 = 1 3 | * * | 5 ⇒ O_1 1 3 2 4 5
ch_2 = 5 3 | * * | 1 O_2 5 3 2 4 1

P_1 : 5 1 | 3 2 | 4 ⇒ * * | 3 2 | 4
P_2 : 4 3 | 1 5 | 2 4 * | 1 5 | *

3 2 | * * | 4 ⇒ O_1 = 3 2 | 1 5 | 4
4 1 | * * | 5 O_2 = 4 1 | 3 2 | 5

P_1 : 5 1 | 3 2 | 4 ⇒ * * | 3 2 | 4
P_2 : 4 3 | 1 5 | 2 4 * | 1 5 | *

3 2 | * * | 4 ⇒ 3 2 | 1 5 | 4
4 1 | * * | 5 4 1 | 3 2 | 5

Order Crossover (OX)

Solve the following TSP using the GA with population size $P=6$,

	1	2	3	4	5
1	–	3	6	2	3
2	3	–	8	1	9
3	5	7	–	2	7
4	4	5	8	–	6
5	2	4	3	7	–

Solution: Notice that $n = 5$ and start from city 1.

Let the $P=6$ chromosomes are as follows:

Initialization:

$ch_1 = [4\ 2\ 5\ 3]$, $ch_2 = [5\ 2\ 4\ 3]$, $ch_3 = [2\ 3\ 4\ 5]$

$ch_4 = [5\ 4\ 2\ 3]$, $ch_5 = [3\ 2\ 5\ 4]$, $ch_6 = [4\ 2\ 3\ 5]$;

Fitness: add city 1 for ch_1 which be $ch_1 = [1\ 4\ 2\ 5\ 3\ 1]$,

then $f_1 = 2+5+9+3+5 = 24$

$f_2 = 23$, $f_3 = 21$, $f_4 = 28$, $f_5 = 33$, $f_6 = 26$

Best fitness is $f_3 = 21$.

Selection: $F = \sum f_i = 155$

$\alpha_i = 0.15, 0.15, 0.14, 0.18, 0.21, 0.17$

we have intervals $[0,15)$, $[15,30)$, $[30,44)$, $[41,62)$

$[62,83)$, $[83,100)$.

(6) number are drawn: 50, 71, 20, 43, 17, 85

Then the following chromosoms are selected:

$ch_4, ch_5, ch_2, ch_3, ch_2, ch_6$

ossover: $P_{crossover} = 0.7$

let $P_c = 0.65$ (change) ≤ 0.7 $ch_4 = [5 | 4 \; 2 | 3]$ ⟹ $ch_1' = [* | 4 \; 2 | 3]$

$ch_5 = [3 | 2 \; 5 | 4]$ $ch_2' = [3 | 2 \; 5 | *]$

$ch_1' = [4 | * \; * | 3]$ ⟹ $ch_1' = [4 | 2 \; 5 | 3]$

$ch_2' = [3 | * \; * | 2]$ $ch_2' = [3 | 4 \; 2 | 5]$

let $P_C = 0.6 \leq 0.7$ (change)

- $ch_2 = [5 | 2 \; 4 | 3]$ ⟹ $ch_3' = [5 | 3 \; 4 | 2]$

$ch_3 = [2 | 3 \; 4 | 5]$ $ch_4' = [3 | 2 \; 4 | 5]$

let $P_c = 0.8 > 0.7$ (no change)

$ch_2 = [5 | 2 \; 4 | 3]$ ⟹ $ch_5' = [5 | 2 \; 4 | 3]$

$ch_6 = [4 | 2 \; 3 | 5]$ $ch_6' = [4 | 2 \; 3 | 5]$

Mutation let $P_M = 0.07$. Swapping

1. let $P_m = 0.08 > 0.07$ (no change) $ch_1 = ch_1' = [4 \; 2 \; 5 \; 3]$
2. let $P_m = 0.06 \leq 0.07$ (change), let $l_k = 2$, then $ch_2 = [3 \; 2 \; 4 \; 5]$
3. let $P_m = 0.05 \leq 0.07$ (change), let $l_k = 3$, then $ch_3 = [5 \; 3 \; 2 \; 4]$
4. let $P_m = 0.1 > 0.07$ (no change), then $ch_4 = [3 \; 2 \; 4 \; 5] = ch_4'$
5. let $P_m = 0.09 > 0.07$ (no change), then $ch_5 = ch_5' = [5 \; 2 \; 4 \; 3]$
6. let $P_m = 0.06 \leq 0.07$ (change), then $l_k = 1$, $ch_6 = [2 \; 4 \; 3 \; 5]$

New Generation: $ch_1 = [4 \; 2 \; 5 \; 3]$, $ch_2 = [3 \; 2 \; 4 \; 5]$, …, $ch_6 = [2 \; 4 \; 3 \; 5]$

Fitness: $f_1 = 24$, $f_2 = 24$, $f_3 = 16$, $f_4 = 22$

$f_5 = 21$, $f_6 = 22$

Best fitness $f_3 = 16$, for Chromosome $ch_3 = [5\ 3\ 4\ 2]$

with path 1-5-3-4-2-1