CHAPTER THREE

AIRCRAFT LANDING PROBLEMS (ALP)

3.1 Introduction

Aircraft Landing Problem (ALP): Given a set of planes in the radar horizon of an air-traffic controller (ATC), the problem is one of determining a landing time at a busy airport for each plane such that each plane in this ATC horizon lands within a prespecified landing time window and such that landing separation criteria specified for each pair of planes in this horizon are adhered to.

In this chapter, we study the aircraft landing problem (ALP), which considered as one of a COPs, in a single runway case. We present in the first part, a mathematical formulation of the problem with a linear objective function. In the second part, we consider the static case of the problem where all data are known in advance. We present a new heuristic for scheduling static case of aircraft landing to solve the ALP.

3.2 Aircraft Landing Problem (ALP)

3.2.1 ALP Motivation

When the number of approaching flights exceeds the airport capacity, some of these aircraft can't be landed on its ′perfect′ landing time. However, some costs are being considered:

- There is a cost mainly on the waste of fuel for each plane flying faster than its most economical speed.
- Airlines also experience different costs for delays of different flights.
- Depending on the amount of delay, there might be a number of transfer passengers that miss their connecting flight.

- The crew or aircraft might also be needed to perform a next flight, which now has to be rescheduled.
- This might propagate delays to departing flights.

3.2.2 Description and Notations of the ALP

Assumptions of ALP are as follows:

- The set of aircrafts which are waiting to be landed is known, a static model.
- There are several runways in the airport.
- The sets of aircrafts including the target time and time window are waiting to be landed on the runway.
- The cost is considered for each unit of tardiness or earliness for the target time of every aircraft.
- Each aircraft is supposed to land on a determined runway, when the limitation of separation time (the time between this aircraft and previous ones which land on this runway or others) is satisfied.
- All aircrafts are not equal and similar to each other and there are different aircrafts.

The **objective function** of the problems is to minimize the deviation of target time for each aircraft. As when an aircraft lands sooner than the target time, it causes problems for other aircrafts flight schedules. Now, we shall assume that we are minimizing total cost, where the cost for any plane is linearly related to deviation from its target time. Figure (3.1) illustrates the variation in cost within the time window of a particular plane.

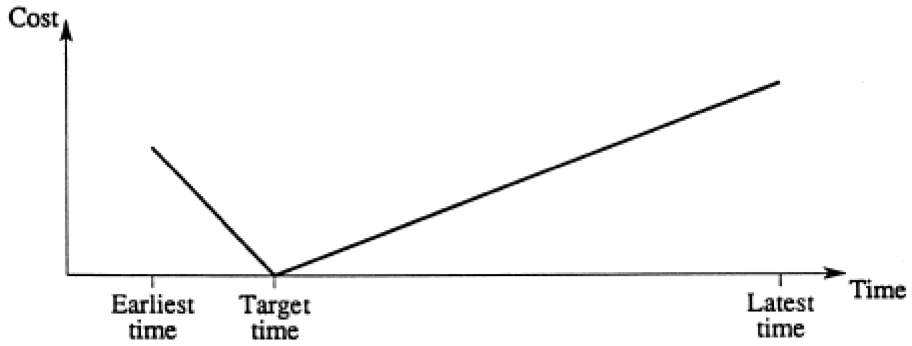

Figure (3.1): Variation in cost for a plane within its time window.

The aircraft generally partitioned into three weight classes: Small, Large and Heavy. The time separation requirements are then a function of the plane speed and the length of the final approach path.

We first observe that the ALP involves two decision problems: (1) a **sequencing** problem (which determines the sequence of plane landings) and (2) a **scheduling** decision problem (which determines the precise landing times for each plane in the sequence, subject to the separation constraints).

The ALP has the following notations:

N : the number of planes.

P : Set of N planes, $P=\{1,2,\ldots,N\}$.

R : the number of landing runways (here we take R=1).

E_i : the earliest landing time for plane $i \in P$.

L_i : the latest landing time for plane $i \in P$.

T_i : the target (preferred) landing time for plane $i \in P$.

S_{ij} : the required separation time (≥ 0) between plane i landing and plane j landing (where plane i lands before plane j), $i,j \in P$, $i \neq j$.

g_i : the penalty cost (≥ 0) per unit of time for landing before the target time T_i for plane $i \in P$.

h_i : the penalty cost (≥ 0) per unit of time for landing after the target time T_i for plane $i \in P$.

The variables are:

t_i : the actual landing time for plane $i \in P$.

α_i : how soon plane $i\in P$ lands before T_i, mathematically, the earliness $\alpha_i=\max\{0,T_i - t_i\}$.

β_i : how soon plane $i\in P$ lands after T_i, mathematically, the tardiness $\beta_i=\max\{0 , t_i - T_i\}$.

$$\delta_{ij} = \begin{cases} 1 \text{ if plane } i \text{ lands before plane } j\ \forall i,j\in P,\ i\neq j. \\ 0 \text{ otherwise.} \end{cases}$$

Without significant loss of generality, we shall henceforth assume that the times E_i, L_i, and S_{ij} are integers.

3.3 Single Runway Formulation of ALP

In this section, we present an initial mixed-integer zero–one formulation of the static single runway ALP.

3.3.1 ALP Constraints

The first set of constraints are

$$E_i \le t_i \le L_i,\ \forall i\in P, \qquad \ldots(3.1)$$

which ensure that each plane lands within its time window. Now, considering pairs (i,j) of planes we have that

$$\delta_{ij}+\delta_{ji}=1,\ \forall i,j\in P,\ i<j, \qquad \ldots(3.2)$$

We need to define three sets:

U : the set of pairs (i,j) of planes for which we are uncertain whether plane i lands before plane j.

V : the set of pairs (i,j) of planes for which i definitely lands before j (but for which the separation constraint is not automatically satisfied).

W : the set of pairs (i,j) of planes for which i definitely lands before j (and for which the separation constraint is automatically satisfied).

Then, we can define the set W by

$$W = \{(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} \le E_j,\ \forall i,j \in P,\ i \ne j]\} \qquad \ldots(3.3)$$

In words, i must land before j ($L_i<E_j$) and the separation constraint is automatically satisfied ($L_i+S_{ij}\le E_j$).

We can define the set V by

$$V = \{(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} > E_j,\ \forall i,j \in P,\ i \ne j\} \qquad \ldots(3.4)$$

In words, i must land before j ($L_i<E_j$) but the separation constraint is not automatically satisfied ($L_i+S_{ij}>E_j$).

Some plane lands first may have overlapping time windows. Hence, we can define the set U as:

$$U=\{(i,j)|i,j=1,\ldots,N, i\ne j; E_j\le E_i\le L_j \text{ or } E_j\le L_i\le L_j,\ E_i\le E_j\le L_i \text{ or } E_i\le L_j\le L_i\} \ldots(3.5)$$

We need a separation constraint for pairs of planes in V, and this is

$$t_j \ge t_i + S_{ij}\ \ \forall (i,j)\in V \qquad \ldots(3.6)$$

which ensures that a time S_{ij} must elapse after the landing of plane i at t_i before plane j can land at t_j.

Finally, we need constraints to relate the α_i, β_i, and t_i variables to each other.

$$T_i - t_i \le \alpha_i \le T_i - E_i,\quad i\in P, \qquad \ldots(3.7)$$

$$t_i - T_i \le \beta_i \le L_i - T_i,\quad i\in P, \qquad \ldots(3.8)$$

3.3.2 Objective Function

First, we have the following objective function:

$$\text{minimize } Z_1 = \sum_{i=1}^{N} t_i \qquad \ldots(3.9)$$

which mean the sum of all actual times of all aircrafts.

We now need only to setup the objective function, minimize the deviation from the target times (T_i), and this is

$$\text{minimize } Z_2 = \sum_{i=1}^{N} (g_i\alpha_i + h_i\beta_i) \qquad \ldots(3.10)$$

Lastly we have the following multi-Criteria Objective function (MCOF):

$$\text{minimize } Z_3 = (Z_1, Z_2) \qquad \ldots(3.11)$$

The complete formulation (model) of the single runway problem is therefore to satisfy function (3.9), (3.10) and (3.11) subject to relations (3.1), (3.2), and (3.6)-(3.8).

Example (3.1): For N=3, lets have the following ALP information.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Formulate this ALP:

$$Z_1 = \sum_{i=1}^{N} t_i$$

$$Z_2 = \sum_{i=1}^{3} (g_i\alpha_i + h_i\beta_i)$$

Z_1=(10*max{0,155-t_1}+10*max{0,t_1-155})+(10*max{0,258-t_2}+ 10*max{0,t_2-258})+(30*max{0,98-t_3}+10*max{0,t_3-98})

s.t.

$129 \le t_1 \le 559$, $195 \le t_2 \le 744$, $89 \le t_3 \le 510$,

$\delta_{ij}+\delta_{ji}=1$, $\forall i,j \in P$, $i<j$,

$t_j \ge t_i + S_{ij}$ $\forall (i,j) \in V$,

$155-t_1 \le \max\{0,155-t_1\} \le 26$, $258-t_2 \le \max\{0,258-t_2\} \le 63$, $98-t_3 \le \max\{0,98-t_1\} \le 9$,

$t_1-155 \le \max\{0,t_1-155\} \le 404$, $t_2-258 \le \max\{0,t_2-258\} \le 486$, $t_3-98 \le \max\{0,t_3-98\} \le 412$.

Calculate the objective functions Z_1 and Z_2 if t_i=150, 250, 100:

Z_1=500, while for Z_2 we have:

$\alpha_i=\max\{0,T_i-t_i\}$, i=1,2,3, then:

$\alpha_i=\max\{0,155-150\}=5$, $\max\{0,258-250\}=8$, $\max\{0,98-100\}=0$.

And $\beta_i=\max\{0, t_i-T_i\}$, i=1,2,3, then:

β_i=max{0,150-155}=0, max{0,250-258}=0, max{0,100-98}=2.

$Z=(g_1\alpha_1+h_1\beta_1)+(g_2\alpha_2+h_2\beta_2)+(g_3\alpha_3+h_3\beta_3)=10\times5+10\times8+30\times2=190$.

Lastly for Z_3, we have $Z_3=(Z_1,Z_2)=(500,190)$.

3.4 Techniques to Improve the Solution and Reduce the Computations

In this section we demonstrate two types of methods which are contribute in improving the solution and speed the approach to the good solution. In addition, we will discuss some special cases of ALP.

3.4.1 Time Window Tightening (TWT)

Let Z_{UB} (for any three objective functions) be any upper bound to the problem. Then, it is possible to limit the deviation from target for each plane. Specifically, for plane i, we can update E_i using:

$$E_i = \max\{E_i, T_i - Z_{UB}/g_i\},\ \ i \in P, \qquad \ldots(3.12)$$

Similarly we have

$$L_i = \min\{L_i, T_i + Z_{UB}/h_i\},\ \ i \in P, \qquad \ldots(3.13)$$

The benefit of tightening (closing) the time windows is that (potentially) the sets U and V can be reduced in size, thereby giving a smaller problem to solve.

Example (3.2): The time window tightening of example (3.1) using Eq. (3.12) and (3.13). for instance, Z_{UB}=1060 (using Z_2) we have:

$E_i=\max\{E_i,T_i-106\}$ where: E_1=max{129,155-106}=129,

E_2=max{195,258-106}=195, E_3=max{89,98-35}=98.

$L_i=\min\{L_i,T_i+106\}$ where: L_1=min{559,155+106}=261,

L_2=min{744,258+106}=364, L_3=min{89,98+35}=133.

These results are shown in table (3.1).

Table (3.1): time window tightening of example (3.2) for Z_{UB}=1060.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	261	364	133
g_i	10	10	30
h_i	10	10	30

Exercise (3.1): calculate the TWT for:

1. from example (3.1), Z_{UB}=900.
2. from Appendix, for N=10, for 1^{st} 5 aircraft, Z_{UB}=90.
3. from Appendix, for N=15, for 1^{st} 5 aircraft, Z_{UB}=90.

3.4.2 Successive Rules (SR)

Reducing the current sequence is done by using several SR's. When, for each i ($i\in P$), and with its cost given in the objective function (3.9), we can derive SR that restrict the search for an optimal solution. Such rules can be used in some optimization algorithms. These improvements lead to very large decrease in the number of solutions to obtain the optimal solution.

Definition (3.1): Let $W_i=[E_i,L_i]$ be the time window interval of plane $i\in P$, if $W_i \cap W_j=\phi$ (time windows are disjoint) and $L_i<E_j$ we denote for the interval W_i precedes the interval W_j in line number by $W_i 3 W_j$.

Definition (3.2): We say that plane i precedes the plane j (we write $i\rightarrow j$ or $(i,j)\in W$) or j precedes the plane i if $W_i \cap W_j=\phi$, for $i\neq j$.

Remark (3.1):

1. $t_i < t_j$ and $t_j\geq t_i+S_{ij}$ if and only if $i\rightarrow j$, $\forall i,j\in P$, $i\neq j$.
2. if $E_i\leq E_j\leq L_i$ or $E_i\leq L_j\leq L_i$, then $W_i\cap W_j\neq\phi$ for $i\neq j$, we say that W_i and W_j are overlapped.

Proposition (3.1): if $W_i 3 W_j$, then $t_i \in W_i < t_j \in W_j$, $\forall i,j \in P$, $i \neq j$.

Proof: since $W_i 3 W_j$, then $t_i \notin W_j$ and $t_j \notin W_i$. Suppose $t_i \geq t_j$, for $t_i = t_j$, $t_j = t_i \in W_i$, C!. For $t_i > t_j$, if $t_j \in W_i$ C!. Take $t_j \notin W_i$. Then $t_j \in$ another interval say W_k, s.t. $W_k 3 W_j$, but $t_j \in W_j$ and that is a contradiction since there is no integer belong to two disjoint intervals in the same time. Then $t_i < t_j$ □

Remark (3.2): if $W_i \cap W_j = \phi$, then $L_i < E_j$ or $L_j < E_i$, $\forall i,j \in P$, $i \neq j$.

Definition (3.3): the $i \rightarrow j$ if one of the following conditions is satisfied:

1. $L_i < E_j$ for $i \neq j$.
2. For $L_i \geq E_j$, if $L_i < E_j + S_{ij}$ for $i \neq j$.

Conditions of SR are shown in figure (5.2).

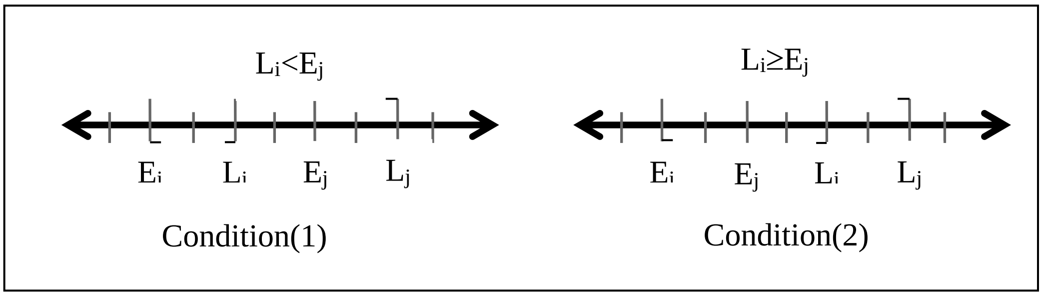

Figure (3.2): Conditions of dominance rules.

Example (3.3): For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	129	89	96	111	123
T_i	155	98	106	123	135
L_i	191	110	118	135	147
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

From definition (3.3), condition (1) we obtain the following SR's:

$2 \rightarrow 1$, $2 \rightarrow 4$, $2 \rightarrow 5$, $3 \rightarrow 1$, $3 \rightarrow 5$.

From condition (2), we have $3 \rightarrow 4$ because of $E_4 + S_{34} = 111 + 8 = 119 > L_3 = 118$, and $4 \rightarrow 1$ because of $E_1 + S_{41} = 129 + 15 = 144 > L_4 = 135$. Figure (3.3) shows the SR's of example (3.3).

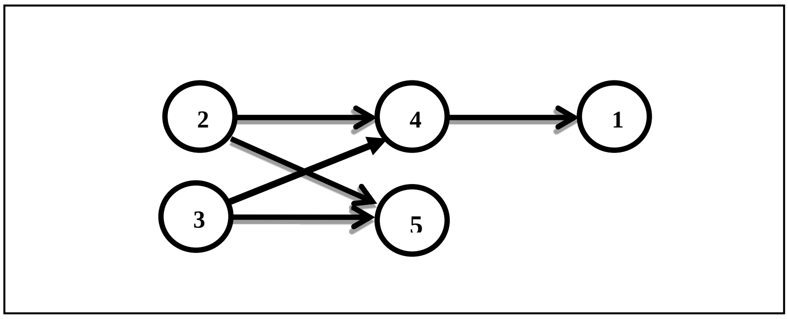

Figure (3.3): Graph of SR of example (3.3).

The adjacency matrix A of the graph shown above is:

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & \delta_{15} \\ 1 & 0 & \delta_{23} & 1 & 1 \\ 1 & \delta_{32} & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \delta_{45} \\ \delta_{51} & 0 & 0 & \delta_{54} & 0 \end{bmatrix}$$

Note:

- $\delta_{15}+\delta_{51}=1$, $\delta_{23}+\delta_{32}=1$, $\delta_{45}+\delta_{54}=1$
- the sequencing problem of this ALP can solved by $2^3=8$ possible and no need to try $5!=120$ possible.

Example (3.4): Find the possible sequences for example (3.3):

From adjacency matrix A, we have $(\delta_{15},\delta_{23},\delta_{45})$, 1↔5,2↔3 and 4↔5.

So we have:

i	$(\delta_{15},\delta_{23},\delta_{45})$	Subsequence	sequence	Acceptance
1.	(0,0,0)	5→1,3→2,5→4	3→2→5→4→1	✓
2.	(0,0,1)	5→1,3→2,4→5	3→2→4→5→1	✓
3.	(0,1,0)	5→1,2→3,5→4	2→3→5→4→1	✓
4.	(0,1,1)	5→1,2→3,4→5	2→3→4→5→1	✓
5.	(1,0,0)	1→5,3→2,5→4	3→2→1→5→4	✗
6.	(1,0,1)	1→5,3→2,4→5	3→2→4→1→5	✓
7.	(1,1,0)	1→5,2→3,5→4	2→3→1→5→4	✗
8.	(1,1,1)	1→5,2→3,4→5	2→3→4→1→5	✓

3.4.3 Special Cases

Definition (3.4): Let $S=\max\{S_{ij}\}$, $\forall i,j\in P, i\neq j$, then W_i is called **logical time window** if the length ℓ_i of W_i, for $i\in P$ is $\ell_i=L_i-E_i+1\geq 2S$ and $T_i=(E_i+L_i)/2$.

Example (3.3): let $W_1=[10,20]$ and $W_2=[25,50]$, $S_{12}=10$, $S=10$. Note that $\ell_1=11$ and $\ell_2=26$, W_2 is logical time window but W_1 is not. While if $W_1=[10,15]$ and $W_2=[16,24]$, $S_{12}=15$, $S=15$. Note that both W_1 and W_2 are not logical time windows, since if $t_1=E_1=10$, then $t_2<t_1+S_{12}=10+15=25>L_2=24$, that mean W_2 is not logical time definitely, not satisfies the separation constraint.

Case (1): Let $W_{i1},W_{i2},\ldots,W_{iN}$ are all disjoint logical time windows in this sequence s.t. $W_{i_k}\cap W_{i_j}=\phi, \forall i_k,i_j\in P$, $i_k\neq i_j$, then the optimal solution with cost Z=0 at $t_{i_1}=T_{i_1}<t_{i_2}=T_{i_2}<\cdots<t_{i_N}=T_{i_N}$ and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: Without loosing the generality, let N=3 to show Z=0 and $1\rightarrow 2\rightarrow 3$. Since W_1,W_2 and $,W_3$ are logical time windows this mean $S=\max\{S_{ij}\}$, $\forall i,j\in P$. Let $t_1=T_1$, $T_1+S\leq L_1<E_2<T_2$, then take:

$$t_2=T_2>T_1+S=t_1+S \qquad \ldots(a)$$

$\therefore$ $t_1=T_1$ and $t_2=T_2$ satisfy the window and separation conditions (WSC's). By applying relation (a) again for t_2 and t_3 we obtain that: $t_2=T_2$ and $t_3=T_3$ satisfy the WSCs.

$\therefore$ The optimal solution with cost Z=0 for N=3 and $1\rightarrow 2\rightarrow 3$.

Consequently, this case can be applied for N aircraft and for any sequence π.□

Case (2): Let $W=W_1=W_2=\ldots=W_N$ be the same large time window, then the optimal solution Z=0 at $t_{i_k}=T_{i_k}$ if T_{i_k} satisfies the separation constraint $\forall i_k\in P$ and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: let's take any arbitrary sequence π. Since T_{i_k} satisfy the separation constraints, this means: $T_1\leq T_2-S_{12}$, $T_2\leq T_3-S_{23},\ldots,$ $T_{N-1}\leq T_N-S_{N-1,N}$. If we take $t_{i_k}=T_{i_k}$, then the landing times t_{i_k} satisfy the separation constraint $\forall i_k\in P$.

$\therefore$ The optimal solution with cost Z=0 and $1 \rightarrow 2 \rightarrow \ldots \rightarrow N$. □

Of course, this case can be applied for any sequence π.

Exercise (3.2): Find the SR for:

1. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	129	111	123	89	96
T_i	155	123	135	98	106
L_i	191	135	147	110	118
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

2. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	146	241	90	95	108
T_i	155	250	93	98	111
L_i	164	259	96	101	114
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

3. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	241	146	108	90	95
T_i	250	155	111	93	98
L_i	259	164	114	96	101
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0