Combinatorial Optimization Problems

4th grade – S & OP Branch/ 2019-2020

Introduced By

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Chapter Three - 1

Aircraft Landing Problems

Introduction

Aircraft Landing Problem (ALP): Given a set of planes in the radar horizon of an air-traffic controller (ATC), the problem is one of determining a landing time at a busy airport for each plane such that each plane in this ATC horizon lands within a prespecified landing time window and such that landing separation criteria specified for each pair of planes in this horizon are adhered to.

Aircraft Landing Problem (ALP)

ALP Motivation

When the number of approaching flights exceeds the airport capacity, some of these aircraft can't be landed on its ’perfect’ landing time. However, some costs are being considered:

- There is a cost mainly on the waste of fuel for each plane flying faster than its most economical speed.
- Airlines also experience different costs for delays of different flights.
- Depending on the amount of delay, there might be a number of transfer passengers that miss their connecting flight.
- The crew or aircraft might also be needed to perform a next flight, which now has to be rescheduled.
- This might propagate delays to departing flights.

Aircraft Landing Problem (ALP)

Description and Notations of the ALP

- The set of aircrafts which are waiting to be landed is known, a static model.
- There are several runways in the airport.
- The sets of aircrafts including the target time and time window are waiting to be landed on the runway.
- The cost is considered for each unit of tardiness or earliness for the target time of every aircraft.
- Each aircraft is supposed to land on a determined runway, when the limitation of separation time (the time between this aircraft and previous ones which land on this runway or others) is satisfied.
- All aircrafts are not equal and similar to each other and there are different aircrafts.

The **objective function** of the problems is to minimize the deviation of target time for each aircraft. As when an aircraft lands sooner than the target time, it causes problems for other aircrafts flight schedules. Now, we shall assume that we are minimizing total cost, where the cost for any plane is linearly related to deviation from its target time

Aircraft Landing Problem (ALP)

- A **sequencing** problem (which determines the sequence of plane landings).
- A **scheduling** decision problem (which determines the precise landing times for each plane in the sequence).

The ALP has the following notations:

N : the number of planes.

P : Set of N planes, $P=\{1,2,...,N\}$.

R : the number of landing runways (here we take $R=1$).

E_i : the earliest landing time for plane $i \in P$.

L_i : the latest landing time for plane $i \in P$.

T_i : the target (preferred) landing time for plane $i \in P$.

S_{ij} : the required separation time (≥ 0) between plane i landing and plane j landing (where plane i lands before plane j), $i,j \in P$, $i \neq j$.

g_i : the penalty cost (≥ 0) per unit of time for landing before the target time T_i for plane $i \in P$.

h_i : the penalty cost (≥ 0) per unit of time for landing after the target time T_i for plane $i \in P$.

The variables are:

t_i : the actual landing time for plane $i \in P$.

α_i : how soon plane $i \in P$ lands before T_i, mathematically, the earliness $\alpha_i = \max\{0, T_i - t_i\}$.

β_i : how soon plane $i \in P$ lands after T_i, mathematically, the tardiness $\beta_i = \max\{0, t_i - T_i\}$.

$$\delta_{ij} = \begin{cases} 1 \text{ if plane } i \text{ lands before plane } j \ \forall i,j \in P,\ i \neq j. \\ 0 \text{ otherwise.} \end{cases}$$

Single Runway Formulation of ALP

ALP Constraints

- $E_i \leq t_i \leq L_i,\ \forall i \in P,$ ...(1)

considering pairs (i,j) of planes we have that

- $\delta_{ij}+\delta_{ji}=1,\ \forall i,j \in P,\ i<j,$ ...(2)

We need a separation constraint for pairs of planes in V, s.t.

- $t_j \geq t_i + S_{ij}\ \forall (i,j) \in V$ ...(3)

Finally, we need constraints to relate the α_i, β_i, and t_i variables s.t.

- $T_i - t_i \leq \alpha_i \leq T_i - E_i,\quad i \in P,$ ...(4)
- $t_i - T_i \leq \beta_i \leq L_i - T_i,\quad i \in P,$ ...(5)

Objective Function

The objective function, minimize the deviation from the target times (T_i), and this is

minimize $\sum_{i=1}^{N}(g_i\alpha_i + h_i\beta_i)$ ...(6)

Single Runway Formulation of ALP

Example(1): For N=3,

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

S_{ij}	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Formulate this ALP: Z= $\sum_{i=1}^{3}(g_i\alpha_i + h_i\beta_i)$

$Z=(10*\max\{0,155-t_1\}+10*\max\{0,t_1-155\})+(10*\max\{0,258-t_2\}+ 10*\max\{0,t_2-258\})+(30*\max\{0,98-t_3\}+10*\max\{0,t_3-98\})$

s.t.

$129 \le t_1 \le 559$, $195 \le t_2 \le 744$, $89 \le t_3 \le 510$,

$\delta_{ij}+\delta_{ji}=1$, $\forall i,j \in P$, $i<j$,

$t_j \ge t_i+S_{ij}$ $\forall (i,j) \in V$,

$155-t_1 \le \max\{0,155-t_1\} \le 26$, $258-t_2 \le \max\{0,258-t_2\} \le 63$, $98-t_3 \le \max\{0,98-t_1\} \le 9$,

$t_1-155 \le \max\{0,t_1-155\} \le 404$, $t_2-258 \le \max\{0,t_2-258\} \le 486$, $t_3-98 \le \max\{0,t_3-98\} \le 412$.

Calculate the objective function Z if t_i=150, 250, 100:

we have $\alpha_i=\max\{0,T_i-t_i\}$, i=1,2,3, then:

$\alpha_i=\max\{0,155-150\}=5$, $\max\{0,258-250\}=8$, $\max\{0,98-100\}=0$.

And $\beta_i=\max\{0, t_i-T_i\}$, i=1,2,3, then:

$\beta_i=\max\{0,150-155\}=0$, $\max\{0,250-258\}=0$, $\max\{0,100-98\}=2$.

$Z=(g_1\alpha_1+h_1\beta_1)+(g_2\alpha_2+h_2\beta_2)+(g_3\alpha_3+h_3\beta_3)=10\times5+10\times8+30\times2=190$.

Techniques to Improve the Solution and Reduce the Computations

Time Window Tightening (TWT)

Let Z_{UB} be any upper bound to the problem. Then, it is possible to limit the deviation from target for each plane. Specifically, for plane i, we can update E_i using:

$E_i = \max\{E_i, T_i - Z_{UB}/g_i\}$, $i \in P$, ...(7)

Similarly we have

$L_i = \min\{L_i, T_i + Z_{UB}/h_i\}$, $i \in P$, ...(8)

The benefit of tightening (closing) the time windows is that reduced in size, thereby giving a smaller problem to solve.

Example (2): The time window tightening of example (1) using Eq. (7) and (8). using Z_{UB}=1060 we have:

$E_i=\max\{E_i,T_i-106\}$ where: $E_1=\max\{129,155-106\}=129$, $E_2=\max\{195,258-106\}=195$, $E_3=\max\{89,98-35\}=98$.

$L_i=\min\{L_i,T_i+106\}$ where: $L_1=\min\{559,155+106\}=261$, $L_2=\min\{744,258+106\}=364$, $L_3=\min\{89,98+35\}=133$.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	261	364	133
g_i	10	10	30
h_i	10	10	30

Techniques to Improve the Solution and Reduce the Computations

Successive Rules (SR)

Definition: Let $W_i=[E_i,L_i]$ be the time window interval of plane $i\in P$, if $W_i \cap W_j=\phi$ (time windows are disjoint) and $L_i<E_j$ we denote for the interval W_i precedes the interval W_j in line number by $W_i \Rightarrow W_j$.

Definition: We say that plane i precedes the plane j (we write $i\rightarrow j$ or $(i,j)\in W$) or j precedes the plane i if $W_i \cap W_j=\phi$, for $i\neq j$.

Remark:

- $t_i < t_j$ and $t_j \geq t_i+S_{ij}$ if and only if $i\rightarrow j$, $\forall i,j\in P$, $i\neq j$.
- if $E_i\leq E_j\leq L_i$ or $E_i\leq L_j\leq L_i$, then $W_i \cap W_j\neq\phi$ for $i\neq j$, we say that W_i and W_j are overlapped.

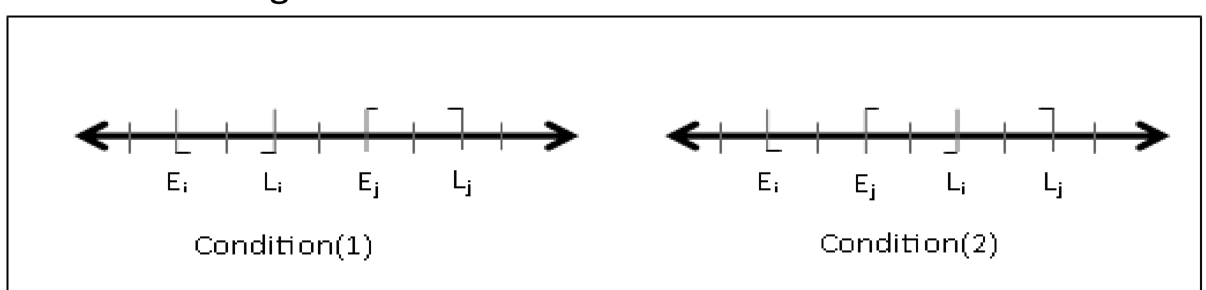
Proposition : if $W_i \Rightarrow W_j$, then $t_i\in W_i<t_j\in W_j$, $\forall i,j\in P$, $i\neq j$.

Proof: since $W_i \Rightarrow W_j$, then $t_i\notin W_j$ and $t_j\notin W_i$. Suppose $t_i\geq t_j$, for $t_i=t_j$, $t_j=t_i\in W_i$, C!. For $t_i>t_j$, if $t_j\in W_i$ C!. Take $t_j\notin W_i$. Then $t_j\in$ another interval say W_k, s.t. $W_k 3 W_j$, but $t_j\in W_j$ and that is a contradiction since there is no integer belong to two disjoint intervals in the same time. Then $t_i<t_j$.

Remark: if $W_i \cap W_j=\phi$, then $L_i < E_j$ or $L_j < E_i$, $\forall i,j\in P$, $i\neq j$.

Definition: the planes $i\rightarrow j$ if one of the following conditions is satisfied:

1. $L_i < E_j$ for $i\neq j$.
2. For $L_i \geq E_j$, if $L_i < E_j+S_{ij}$ for $i\neq j$.



Techniques to Improve the Solution and Reduce the Computations

Example: For N=5:

From definition

condition (1)

we obtain the following SR's:

2→1, 2→4, 2→5, 3→1, 3→5.

From condition (2),

we have

3→4 because of

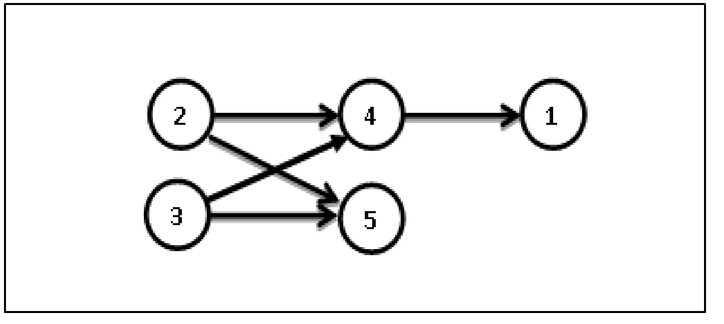
$E_4+S_{34}=111+8=119 > L_3=118$,

and 4→1 because of

$E_1+S_{41}=129+15=144 > L_4=135$.

	P_1	P_2	P_3	P_4	P_5
E_i	129	89	96	111	123
T_i	155	98	106	123	135
L_i	191	110	118	135	147
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0



Techniques to Improve the Solution and Reduce the Computations

The adjacency matrix A of the graph shown above is:

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & \delta_{15} \\ 1 & 0 & \delta_{23} & 1 & 1 \\ 1 & \delta_{32} & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \delta_{45} \\ \delta_{51} & 0 & 0 & \delta_{54} & 0 \end{bmatrix} \end{array}$$

$\delta_{15}+\delta_{51}=1$, $\delta_{23}+\delta_{32}=1$, $\delta_{45}+\delta_{54}=1$

the sequencing problem of this ALP can solved by $2^3=8$ possible and no need to try $5!=120$ possible.

Find the possible sequences. From matrix A, we have $(\delta_{15},\delta_{23},\delta_{45})$, $1\leftrightarrow5$, $2\leftrightarrow3$ and $4\leftrightarrow5$.

So we have:

i	$(\delta_{15},\delta_{23},\delta_{45})$	Subsequence	sequence	Acceptance
1.	(0,0,0)	$5\to1, 3\to2, 5\to4$	$3\to2\to5\to4\to1$	✓
2.	(0,0,1)	$5\to1, 3\to2, 4\to5$	$3\to2\to4\to5\to1$	✓
3.	(0,1,0)	$5\to1, 2\to3, 5\to4$	$2\to3\to5\to4\to1$	✓
4.	(0,1,1)	$5\to1, 2\to3, 4\to5$	$2\to3\to4\to5\to1$	✓
5.	(1,0,0)	$1\to5, 3\to2, 5\to4$	$3\to2\to1\to5\to4$	×
6.	(1,0,1)	$1\to5, 3\to2, 4\to5$	$3\to2\to4\to1\to5$	✓
7.	(1,1,0)	$1\to5, 2\to3, 5\to4$	$2\to3\to1\to5\to4$	×
8.	(1,1,1)	$1\to5, 2\to3, 4\to5$	$2\to3\to4\to1\to5$	✓

Special Cases of ALP

Definition: Let $S=\max\{S_{ij}\}$, $\forall i,j\in P, i\neq j$, then W_i is called **logical time window (LTW)** if the length l_i of W_i, for $i\in P$ is $l_i=L_i-E_i+1\geq 2S$ and $T_i=(E_i+L_i)/2$.

Example: let $W_1=[10,20]$ and $W_2=[25,50]$, $S_{12}=10$, $S=10$. Note that $l_1=11$ and $l_2=26$, W_2 is LTW but W_1 is not. While if $W_1=[10,15]$ and $W_2=[16,24]$, $S_{12}=15$, $S=15$. Note that both W_1 and W_2 are not LTWs, since if $t_1=E_1=10$, then $t_2<t_1+S_{12}=10+15=25>L_2=24$, that mean W_2 is not LTW definitely, not satisfies the separation constraint.

Case (1): Let $W_{i1},W_{i2},\ldots,W_{iN}$ are all disjoint LTWs in this sequence s.t. $W_{i_k}\cap W_{i_j}=\phi$,$\forall i_k,i_j\in P$, $i_k\neq i_j$, then the optimal solution with cost Z=0 at and $i_1\to i_2\to\ldots\to i_N$.

Proof: Without loosing the generality, let N=3 to show Z=0 and $1\to 2\to 3$.

Since W_1,W_2 and ,W_3 are LTWs this mean $S=\max\{S_{ij}\}$, $\forall i,j\in P$. Let $t_1=T_1$, $T_1+S\leq L_1<E_2<T_2$, then take:

$t_2 = T_2 > T_1+S=t_1+S$ ...(a)

$\therefore$ $t_1=T_1$ and $t_2=T_2$ satisfy the window and separation conditions (WSC's). By applying relation (a) again for t_2 and t_3 we obtain that: $t_2=T_2$ and $t_3=T_3$ satisfy the WSCs.

$\therefore$ The optimal solution with cost Z=0 for N=3 and $1\to 2\to 3$.

Consequently, this case can be applied for N aircraft and for any sequence π. □

Case (2): Let $W=W_1=W_2=\ldots=W_N$ be the same large time window, then the optimal solution Z=0 at $t_{i_k}=T_{i_k}$ if T_{i_k} satisfies the separation constraint $\forall i_k\in P$ and $i_1\to i_2\to\ldots\to i_N$.

Proof: let's take any arbitrary sequence π. Since satisfy the separation constraints, this means: $T_1\leq T_2-S_{12}$, $T_2\leq T_3-S_{23}$,…, $T_{N-1}\leq T_N-S_{N-1,N}$. If we take $t_{i_k}=T_{i_k}$, then the landing times satisfy the separation constraint $\forall i_k\in P$.

$\therefore$ The optimal solution with cost Z=0 and $1\to 2\to\ldots\to N$. □

Special Cases of ALP

Example:

Case (1)

Let N=3, Notice that

$W_i \cap W_j = \phi, \forall i,j$

$t_i = T_i, \forall i$

$\therefore Z=0$ and $3 \to 2 \to 1$.

	P_1	P_2	P_3
E_i	130	124	96
T_i	132	126	98
L_i	134	128	100
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0

Case (2)

Let N=3, Notice that

$W1 = W2 = W3$

$t_i = T_i, \forall i$

$\therefore Z=0$ and $3 \to 2 \to 1$.

	P_1	P_2	P_3
E_i	96	96	96
T_i	131	128	97
L_i	132	132	132
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0

Solving ALP using Heuristic and CE Methods

Parallel Improving Technqiue

The order between aircraft (**sequencing the aircraft**) is setup according to priority rules which are based on the variables:

- **E_i**: The priority is given to the aircraft which has the sooner earliest landing time.
- **T_i** : The priority is given to the aircraft which has the earliest target landing time.
- **L_i** : The priority is given to the aircraft which has the earliest latest landing time.
- **E_i/g_i** : The priority is given to the aircraft which has the soonest earliest time.
- **L_i/h_i** : The priority is given to the aircraft which has the soonest latest time.
- **Ti /(g_i+h_i)**: The priority is given to the aircraft which has the soonest target.
- **1/(g_i+h_i)**: The priority is given to the aircraft which causes the most important advance and lateness penalty.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Example : Let N=3

We have the following priority rules:

E_i: we have the sequence 3,1,2.

T_i: we have the sequence 3,1,2.

L_i: we have the sequence 3,1,2.

E_i/g_i=(12.9,19,5,2.97), we have the sequence 3,1,2.

L_i/h_i=(55.9,74.9,17), we have the sequence 3,1,2.

$T_i/(g_i+h_i)$=(7.75,12.9,1.63), we have the sequence 3,1,2.

$1/(g_i+h_i)$=(0.05,0.05,0.03), we have the sequence 3,1,2 or 3,2,1.

Solving ALP using Heuristic and CE Methods

Parallel Improving Technqiue

The adjusting landing time (scheduling aircraft)

Parallel Improving Algorithm (PIA)

Let P be the list of aircraft set up according to a priority rule and O={}.

1. $t_{P1} \leftarrow T_{P1}$; $P_1 \in O$.

2. **FOR** i = 2 : N

$t_{Pi} \leftarrow \max(T_{Pi}, \max_{Pj \in O}(t_{Pj} + S_{Pi,Pj}))$

END {FOR i}

3. **REPEAT**

Calculate penalty Cost Z

IF ($t_{Pi} > T_{Pi}$)

Reduce the landing time by 1 unit of time

ELSE { $t_{Pi} \leq T_{Pi}$}

Increase the landing time by 1 unit of time

END {IF}

IF (the solution is unfeasible)

Reject the change and keep the last feasible solution.

BREAK.

END {IF}

UNTIL (there is increase of penalty cost)

Parallel Improving Technqiue

Example: For N=10

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
E_i	129	195	89	96	110	120	124
T_i	155	258	98	106	123	135	138
L_i	559	744	510	521	555	576	577
g_i	10	10	30	30	30	30	30
h_i	10	10	30	30	30	30	30

S_{ij}	1	2	3	4	5	6	7
1	0	3	15	15	15	15	15
2	3	0	15	15	15	15	15
3	15	15	0	8	8	8	8
4	15	15	8	0	8	8	8
5	15	15	8	8	0	8	8
6	15	15	8	8	8	0	8
7	15	15	8	8	8	8	0

Suppose that the priority rule is the Ti. The order is as follows:

P_i	3	4	5	6	7	1	2

assign landing time to the 1st aircraft in the list (P_1=3) : $t_3 = T_3$=98, O={3}, then:

P_i	3	4	5	6	7	1	2	Z
t_i	98							0

For the 2nd aircraft in the list (P_2=4), then

$t_4 \leftarrow \max(T_4, \max_{Pj\in O}(t_3 + S_{3,4}))$=max(106,max(98+8))=106, O={3,4}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106						0

For the 3rd aircraft in the list (P_3=5), then

$t_5 \leftarrow \max(T_5, \max(t_3+S_{3,5},t_4+S_{4,5}))$=max(123,max(98+8,106+8))=123, O={3,4,5}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123					0

Parallel Improving Technqiue

Continue example

For the 4th aircraft in the list (P_4=6), then t_6=max(135,max(98+8,106+8,123+8))=135, O={3,4,5,6}, Z=0.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135				0

For the 5th aircraft in the list (P_5=7), then t_7=max(138,max(98+8,106+8,123+8,135+8))=143≠138, O={3,4,5,6,7},

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135	143			150

here we need adjusting the landing time t_7=142, then t_6=134:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	134	142			150

And continue in decreasing until we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139			150

If we continue another step we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	122	130	138			180

Since Z=180, we ignore this step and back to the last step when Z=150.

Parallel Improving Technqiue

Continue example

For the 6^{th} aircraft in the list (P_6=1), then

T_1=max(155,max(98+15,106+15,123+15,131+15,139+15))=155,O={3,4,5,6,7,1}, now we need no adjusting the landing time so we obtain, Z=700

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155		150

For the 7th aircraft in the list (P7=2), then

T2=max(258,max(98+15,106+15,123+15,131+15,139+15,155+15))=258,O={3,4,5,6,7,1,2}, now we need no adjusting the landing time so we obtain, Z=150:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155	258	150

This Table shows the implementation of PIA for this example.

Stage	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**Cost Z**
1	98							0
2	98	106						0
3	98	106	123					0
4	98	106	123	135				0
5	98	106	123	131	139			150
6	98	106	118	131	139	155		150
7	98	106	118	131	139	155	258	150

Complete Enumeration Method (CEM)

When using CEM, in sequencing stage we will try all the possible permutation of N planes which equal to N!, while in scheduling stage we will apply two methods:

- **Exhaustive Search Method (ESM** we try all possibilities starting from E_i ending in L_i. The total number of all possibilities for scheduling is $\prod_{i=1}^{N}(L_i - E_i + 1)$
- **PIA.**

the total complexity (C(N)) for sequencing and scheduling using CEM is:

$$C(N)=N!*\prod_{i=1}^{N}(L_i - E_i + 1) \qquad \ldots(8)$$

For $E_i=T_i=L_i$, $(Z=0)$ $\forall i \in P$, then $C(N)=N!$.

Remark: In general, if :

R: the number of pairs of aircraft which are satisfy SR's.

D: the number of pairs of aircraft which are not submitted to SR's, represented by the variables δ_{ij} in matrix A.

$R+D=C_2^N=N*(N-1)/2$, for the ALP we have 2^D sequences can be try to find the best sequence. In some ALP, N! may be larger than 2^D and vice versa.

N	C(N)	N!	C_2^N	R	D	2^D
8	9.689287×10^{16}	40320	28	17	11	2048*
9	2.486859×10^{20}	362880*	36	17	19	524288
10	1.892271×10^{23}	3628800*	45	23	22	4194304
15	5.084773×10^{41}	1.3077×10^{12}*	105	44	61	2.305843×10^{18}

Complete Enumeration Method (CEM)

Example : N=3

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	133	130	99
g_i	10	10	30
h_i	10	10	30

S_{ij}	1	2	3
1	0	4	4
2	4	0	4
3	4	4	0

CEM-ESM

The general Complexity is C(3)=6*64=384.The number of SR=3, R=3 and D=0 so we have the unique sequence π=(3,2,1), then C(3) reduces to 64 possible. Then the best solutions using CEM-ESM are:

1 - 96,127,131, Z=40.

2 - 96,128,132, Z=40.

3 - 97,127,131, Z=10.

4 - 97,128,132, Z=10.

CEM-PIA

we have the unique sequence π=(3,2,1), then the best solution using CEM-PIA is:

1 - 97,127,131, Z=10.

Data Set

Table (1)

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Table(2-1)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
E_i	129	195	89	96	110	120	124	126	135	160
T_i	155	258	98	106	123	135	138	140	150	180
L_i	559	744	510	521	555	576	577	573	591	657
g_i	10	10	30	30	30	30	30	30	30	30
h_i	10	10	30	30	30	30	30	30	30	30

Table(2-2)

S_{ij}	1	2	3	4	5	6	7	8	9	10
1	0	3	15	15	15	15	15	15	15	15
2	3	0	15	15	15	15	15	15	15	15
3	15	15	0	8	8	8	8	8	8	8
4	15	15	8	0	8	8	8	8	8	8
5	15	15	8	8	0	8	8	8	8	8
6	15	15	8	8	8	0	8	8	8	8
7	15	15	8	8	8	8	0	8	8	8
8	15	15	8	8	8	8	8	0	8	8
9	15	15	8	8	8	8	8	8	0	8
10	15	15	8	8	8	8	8	8	8	0

Exercises

1. Calculate the TWT for:

- from Table (1), Z_{UB}=900.
- Table(2-1) and table (2-2), for N=10, for 1st 5 aircraft, Z_{UB}=90.

2. Find the SR for N=5

	P_1	P_2	P_3	P_4	P_5
E_i	129	111	123	89	96
T_i	155	123	135	98	106
L_i	191	135	147	110	118
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

	P_1	P_2	P_3	P_4	P_5
E_i	146	241	90	95	108
T_i	155	250	93	98	111
L_i	164	259	96	101	114
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

3. Find the priority rules for Exercise (2).

Exercises

4. Apply PIA using T_i priority for N=5

	P_1	P_2	P_3	P_4	P_5
E_i	129	190	84	89	100
T_i	155	250	93	98	111
L_i	305	400	143	148	161
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

	P_1	P_2	P_3	P_4	P_5
E_i	146	249	95	103	120
T_i	155	258	98	106	123
L_i	164	267	101	109	126
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

5. Find C(N), The number of SR, R, D , the possible sequences π, then find the optimal solution for the following ALP using CEM-ESM and CEM-PIA

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	132	129	98
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0