Combinatorial Optimization Problems

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Introduced By

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Chapter Three - 1

Aircraft Landing Problems

Introduction

Aircraft Landing Problem (ALP): Given a set of planes in the radar horizon of an air-traffic controller (ATC), the problem is one of determining a landing time at a busy airport for each plane such that each plane in this ATC horizon lands within a prespecified landing time window and such that landing separation criteria specified for each pair of planes in this horizon are adhered to.

Aircraft Landing Problem (ALP)

ALP Motivation

When the number of approaching flights exceeds the airport capacity, some of these aircraft can't be landed on its ’perfect’ landing time. However, some costs are being considered:

- There is a cost mainly on the waste of fuel for each plane flying faster than its most economical speed.
- Airlines also experience different costs for delays of different flights.
- Depending on the amount of delay, there might be a number of transfer passengers that miss their connecting flight.
- The crew or aircraft might also be needed to perform a next flight, which now has to be rescheduled.
- This might propagate delays to departing flights.

Aircraft Landing Problem (ALP)

Description of the ALP

- The set of aircrafts which are waiting to be landed is known, a static model.
- There are several runways in the airport.
- The sets of aircrafts including the target time and time window are waiting to be landed on the runway.
- The cost is considered for each unit of tardiness or earliness for the target time of every aircraft.
- Each aircraft is supposed to land on a determined runway, when the limitation of separation time (is satisfied.
- All aircrafts are not equal and similar to each other and there are different aircrafts.

The **objective function** of the problems is to minimize the deviation of target time for each aircraft. As when an aircraft lands sooner than the target time, it causes problems for other aircrafts flight schedules. Now, we shall assume that we are minimizing total cost, where the cost for any plane is linearly related to deviation from its target time

Aircraft Landing Problem (ALP)

- A **sequencing** problem (which determines the sequence of plane landings).
- A **scheduling** decision problem (which determines the precise landing times for each plane in the sequence).

The ALP has the following notations:

N : the number of planes.

P : Set of N planes, $P=\{1,2,\dots,N\}$.

R : the number of landing runways (here we take $R=1$).

E_i : the earliest landing time for plane $i \in P$.

L_i : the latest landing time for plane $i \in P$.

T_i : the target (preferred) landing time for plane $i \in P$.

S_{ij} : the required separation time (≥ 0) between plane i landing and plane j landing (where plane i lands before plane j), $i,j \in P$, $i \neq j$.

g_i : the penalty cost (≥ 0) per unit of time for landing before the target time T_i for plane $i \in P$.

h_i : the penalty cost (≥ 0) per unit of time for landing after the target time T_i for plane $i \in P$.

The variables are:

t_i : the actual landing time for plane $i \in P$.

α_i : how soon plane $i \in P$ lands before T_i, mathematically, the earliness $\alpha_i = \max\{0, T_i - t_i\}$.

β_i : how soon plane $i \in P$ lands after T_i, mathematically, the tardiness $\beta_i = \max\{0, t_i - T_i\}$.

$$\delta_{ij} = \begin{cases} 1 \text{ if plane i lands before plane j } \forall i,j \in P,\ i \neq j. \\ 0 \text{ otherwise.} \end{cases}$$

Single Runway Formulation of ALP

ALP Constraints

- $E_i \leq t_i \leq L_i, \ \forall i \in P,$ ...(1)

considering pairs (i,j) of planes we have that

- $\delta_{ij} + \delta_{ji} = 1, \ \forall i,j \in P, \ i<j,$ ...(2)

We need a separation constraint for pairs of planes in V, s.t.

- $t_j \geq t_i + S_{ij} \ \forall (i,j) \in V$ ...(3)

Finally, we need constraints to relate the α_i, β_i, and t_i variables s.t.

- $T_i - t_i \leq \alpha_i \leq T_i - E_i, \quad i \in P,$ ...(4)
- $t_i - T_i \leq \beta_i \leq L_i - T_i, \quad i \in P,$ ...(5)

Objective Function

The objective function, minimize the deviation from the target times (T_i), and this is

minimize $\sum_{i=1}^{N} (g_i \alpha_i + h_i \beta_i)$ ...(6)

Single Runway Formulation of ALP

Example(1): For N=3,

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

S_{ij}	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Formulate this ALP: Z= $\sum_{i=1}^{3}(g_i\alpha_i + h_i\beta_i)$

Z=(10*max{0,155-t_1}+10*max{0,t_1-155})+(10*max{0,258-t_2}+ 10*max{0,t_2-258})+(30*max{0,98-t_3}+10*max{0,t_3-98})

s.t.

$129 \le t_1 \le 559$, $195 \le t_2 \le 744$, $89 \le t_3 \le 510$,

$\delta_{ij}+\delta_{ji}=1$, $\forall i,j \in P$, $i<j$,

$t_j \ge t_i + S_{ij}$ $\forall (i,j) \in V$,

$155-t_1 \le \max\{0,155-t_1\} \le 26$, $258-t_2 \le \max\{0,258-t_2\} \le 63$, $98-t_3 \le \max\{0,98-t_1\} \le 9$,

$t_1-155 \le \max\{0,t_1-155\} \le 404$, $t_2-258 \le \max\{0,t_2-258\} \le 486$, $t_3-98 \le \max\{0,t_3-98\} \le 412$.

Calculate the objective function Z if t_i=150, 250, 100:

we have $\alpha_i=\max\{0,T_i-t_i\}$, i=1,2,3, then:

$\alpha_i=\max\{0,155-150\}=5$, $\max\{0,258-250\}=8$, $\max\{0,98-100\}=0$.

And $\beta_i=\max\{0, t_i-T_i\}$, i=1,2,3, then:

$\beta_i=\max\{0,150-155\}=0$, $\max\{0,250-258\}=0$, $\max\{0,100-98\}=2$.

$Z=(g_1\alpha_1+h_1\beta_1)+(g_2\alpha_2+h_2\beta_2)+(g_3\alpha_3+h_3\beta_3)=10\times5+10\times8+30\times2=190$.