Combinatorial Optimization Problems

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Introduced By

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Chapter Three - 2

Aircraft Landing Problems

Techniques to Improve the Solution and Reduce the Computations

Time Window Tightening (TWT)

Let Z_{UB} be any upper bound to the problem. Then, it is possible to limit the deviation from target for each plane. Specifically, for plane i, we can update E_i using:

$E_i = \max\{E_i, T_i - Z_{UB}/g_i\}, \ i \in P,$ ...(7)

Similarly we have

$L_i = \min\{L_i, T_i + Z_{UB}/h_i\}, \ i \in P,$ ...(8)

The benefit of tightening (closing) the time windows is that reduced in size, thereby giving a smaller problem to solve.

Example (2): The time window tightening of example (1) using Eq. (7) and (8). using Z_{UB}=1060 we have:

$E_i=\max\{E_i,T_i-106\}$ where: $E_1=\max\{129,155-106\}=129$, $E_2=\max\{195,258-106\}=195$, $E_3=\max\{89,98-35\}=98$.

$L_i=\min\{L_i,T_i+106\}$ where: $L_1=\min\{559,155+106\}=261$, $L_2=\min\{744,258+106\}=364$, $L_3=\min\{89,98+35\}=133$.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	261	364	133
g_i	10	10	30
h_i	10	10	30

Techniques to Improve the Solution and Reduce the Computations

Successive Rules (SR)

Definition: Let $W_i=[E_i,L_i]$ be the time window interval of plane $i\in P$, if $W_i \cap W_j=\phi$ (time windows are disjoint) and $L_i<E_j$ we denote for the interval W_i precedes the interval W_j in line number by $W_i \Rightarrow W_j$.

Definition: We say that plane i precedes the plane j (we write $i\rightarrow j$ or $(i,j)\in W$) or j precedes the plane i if $W_i \cap W_j=\phi$, for $i\neq j$.

Remark:

- $t_i < t_j$ and $t_j\geq t_i+S_{ij}$ if and only if $i\rightarrow j$, $\forall i,j\in P$, $i\neq j$.
- if $E_i\leq E_j\leq L_i$ or $E_i\leq L_j\leq L_i$, then $W_i\cap W_j\neq\phi$ for $i\neq j$, we say that W_i and W_j are overlapped.

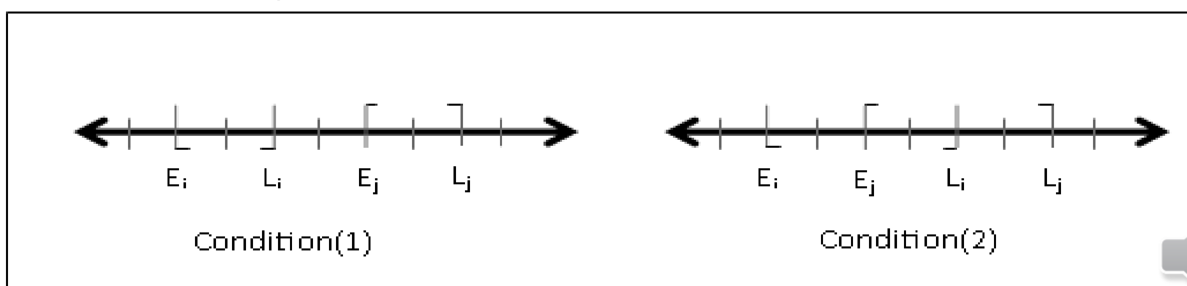
Proposition : if $W_i \Rightarrow W_j$, then $t_i\in W_i<t_j\in W_j$, $\forall i,j\in P$, $i\neq j$.

Proof: since $W_i \Rightarrow W_j$, then $t_i\notin W_j$ and $t_j\notin W_i$. Suppose $t_i\geq t_j$, for $t_i=t_j$, $t_j=t_i\in W_i$, C!. For $t_i>t_j$, if $t_j\in W_i$ C!. Take $t_j\notin W_i$. Then $t_j\in$ another interval say W_k, s.t. $W_k\Rightarrow W_j$, but $t_j\in W_j$ and that is a contradiction since there is no integer belong to two disjoint intervals in the same time. Then $t_i<t_j$.

Remark: if $W_i \cap W_j=\phi$, then $L_i < E_j$ or $L_j < E_i$, $\forall i,j\in P$, $i\neq j$.

Definition: the planes $i\rightarrow j$ if one of the following conditions is satisfied:

1. $L_i < E_j$ for $i\neq j$.
2. For $L_i \geq E_j$, if $L_i < E_j+S_{ij}$ for $i\neq j$.



Techniques to Improve the Solution and Reduce the Computations

Example: For N=5:

From definition condition (1)

we obtain the following SR's:

2→1, 2→4, 2→5, 3→1, 3→5.

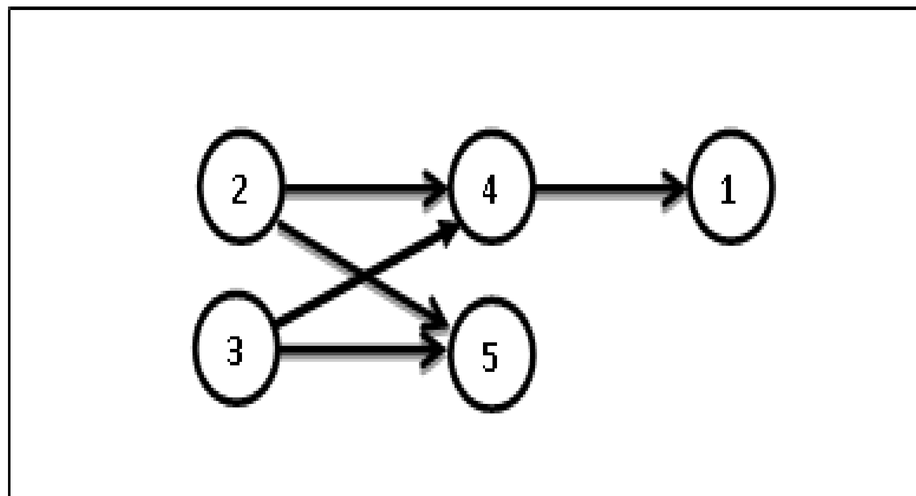
From condition (2), we have

3→4 because of $E_4+S_{34}=111+8=119 > L_3=118$,

and 4→1 because of $E_1+S_{41}=129+15=144 > L_4=135$.

	P_1	P_2	P_3	P_4	P_5
E_i	129	89	96	111	123
T_i	155	98	106	123	135
L_i	191	110	118	135	147
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0



Techniques to Improve the Solution and Reduce the Computations

The adjacency matrix A of the graph shown above is:

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & \delta_{15} \\ 1 & 0 & \delta_{23} & 1 & 1 \\ 1 & \delta_{32} & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \delta_{45} \\ \delta_{51} & 0 & 0 & \delta_{54} & 0 \end{bmatrix} \end{array}$$

$\delta_{15}+\delta_{51}=1$, $\delta_{23}+\delta_{32}=1$, $\delta_{45}+\delta_{54}=1$

the sequencing problem of this ALP can solved by $2^3=8$ possible and no need to try $5!=120$ possible.

Find the possible sequences. From matrix A, we have $(\delta_{15},\delta_{23},\delta_{45})$, $1\leftrightarrow5, 2\leftrightarrow3$ and $4\leftrightarrow5$.

So we have:

i	$(\delta_{15},\delta_{23},\delta_{45})$	Subsequence	sequence	Acceptance
1.	(0,0,0)	5→1,3→2,5→4	3→2→5→4→1	✓
2.	(0,0,1)	5→1,3→2,4→5	3→2→4→5→1	✓
3.	(0,1,0)	5→1,2→3,5→4	2→3→5→4→1	✓
4.	(0,1,1)	5→1,2→3,4→5	2→3→4→5→1	✓
5.	(1,0,0)	1→5,3→2,5→4	3→2→1→5→4	×
6.	(1,0,1)	1→5,3→2,4→5	3→2→4→1→5	✓
7.	(1,1,0)	1→5,2→3,5→4	2→3→1→5→4	×
8.	(1,1,1)	1→5,2→3,4→5	2→3→4→1→5	✓

Special Cases of ALP

Definition: Let $S=\max\{S_{ij}\}$, $\forall i,j\in P, i\neq j$, then W_i is called **logical time window (LTW)** if the length l_i of W_i, for $i\in P$ is $l_i=L_i-E_i+1\geq 2S$ and $T_i=(E_i+L_i)/2$.

Example: let $W_1=[10,20]$ and $W_2=[25,50]$, $S_{12}=10$, $S=10$. Note that $l_1=11$ and $l_2=26$, W_2 is LTW but W_1 is not. While if $W_1=[10,15]$ and $W_2=[16,24]$, $S_{12}=15$, $S=15$. Note that both W_1 and W_2 are not LTWs, since if $t_1=E_1=10$, then $t_2<t_1+S_{12}=10+15=25>L_2=24$, that mean W_2 is not LTW definitely, not satisfies the separation constraint.

Case (1): Let $W_{i1},W_{i2},\ldots,W_{iN}$ are all disjoint LTWs in this sequence s.t. $W_{i_k}\cap W_{i_j}=\phi$,$\forall i_k,i_j\in P$, $i_k\neq i_j$, then the optimal solution with cost Z=0 at and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: Without loosing the generality, let N=3 to show Z=0 and $1\rightarrow 2\rightarrow 3$.

Since W_1,W_2 and ,W_3 are LTWs this mean $S=\max\{S_{ij}\}$, $\forall i,j\in P$. Let $t_1=T_1$, $T_1+S\leq L_1<E_2<T_2$, then take:

$t_2=T_2>T_1+S=t_1+S$ ...(a)

$\therefore$ $t_1=T_1$ and $t_2=T_2$ satisfy the window and separation conditions (WSC's). By applying relation (a) again for t_2 and t_3 we obtain that: $t_2=T_2$ and $t_3=T_3$ satisfy the WSCs.

$\therefore$ The optimal solution with cost Z=0 for N=3 and $1\rightarrow 2\rightarrow 3$.

Consequently, this case can be applied for N aircraft and for any sequence π. □

Case (2): Let $W=W_1=W_2=\ldots=W_N$ be the same large time window, then the optimal solution Z=0 at $t_{i_k}=T_{i_k}$ if T_{i_k} satisfies the separation constraint $\forall i_k\in P$ and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: let's take any arbitrary sequence π. Since satisfy the separation constraints, this means: $T_1\leq T_2-S_{12}$, $T_2\leq T_3-S_{23},\ldots, T_{N-1}\leq T_N-S_{N-1,N}$. If we take $t_{i_k}=T_{i_k}$, then the landing times satisfy the separation constraint $\forall i_k\in P$.

$\therefore$ The optimal solution with cost Z=0 and $1\rightarrow 2\rightarrow\ldots\rightarrow N$. □

Special Cases of ALP

Example:

Case (1)

Let N=3, Notice that

$W_i \cap W_j = \phi,\ \forall i,j$

$t_i = T_i,\ \forall i$

$\therefore Z=0$ and $3 \to 2 \to 1$.

	P_1	P_2	P_3
E_i	130	124	96
T_i	132	126	98
L_i	134	128	100
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0

Case (2)

Let N=3, Notice that

W1=W2 =W3

$t_i = T_i,\ \forall i$

$\therefore Z=0$ and $3 \to 2 \to 1$.

	P_1	P_2	P_3
E_i	96	96	96
T_i	131	128	97
L_i	132	132	132
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0