CHAPTER THREE

AIRCRAFT LANDING PROBLEMS (ALP)

3.1 Introduction

Aircraft Landing Problem (ALP): Given a set of planes in the radar horizon of an air-traffic controller (ATC), the problem is one of determining a landing time at a busy airport for each plane such that each plane in this ATC horizon lands within a prespecified landing time window and such that landing separation criteria specified for each pair of planes in this horizon are adhered to.

In this chapter, we study the aircraft landing problem (ALP), which considered as one of a COPs, in a single runway case. We present in the first part, a mathematical formulation of the problem with a linear objective function. In the second part, we consider the static case of the problem where all data are known in advance. We present a new heuristic for scheduling static case of aircraft landing to solve the ALP.

3.2 Aircraft Landing Problem (ALP)

3.2.1 ALP Motivation

When the number of approaching flights exceeds the airport capacity, some of these aircraft can't be landed on its ′perfect′ landing time. However, some costs are being considered:

- There is a cost mainly on the waste of fuel for each plane flying faster than its most economical speed.
- Airlines also experience different costs for delays of different flights.
- Depending on the amount of delay, there might be a number of transfer passengers that miss their connecting flight.

- The crew or aircraft might also be needed to perform a next flight, which now has to be rescheduled.
- This might propagate delays to departing flights.

3.2.2 Description and Notations of the ALP

Assumptions of ALP are as follows:

- The set of aircrafts which are waiting to be landed is known, a static model.
- There are several runways in the airport.
- The sets of aircrafts including the target time and time window are waiting to be landed on the runway.
- The cost is considered for each unit of tardiness or earliness for the target time of every aircraft.
- Each aircraft is supposed to land on a determined runway, when the limitation of separation time (the time between this aircraft and previous ones which land on this runway or others) is satisfied.
- All aircrafts are not equal and similar to each other and there are different aircrafts.

The **objective function** of the problems is to minimize the deviation of target time for each aircraft. As when an aircraft lands sooner than the target time, it causes problems for other aircrafts flight schedules. Now, we shall assume that we are minimizing total cost, where the cost for any plane is linearly related to deviation from its target time. Figure (3.1) illustrates the variation in cost within the time window of a particular plane.

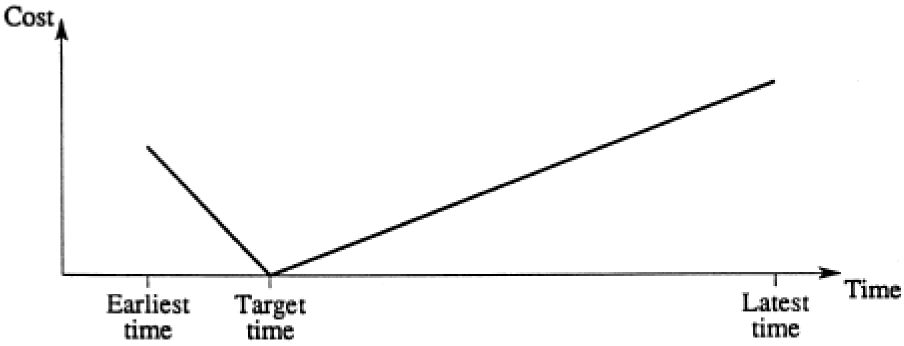

Figure (3.1): Variation in cost for a plane within its time window.

The aircraft generally partitioned into three weight classes: Small, Large and Heavy. The time separation requirements are then a function of the plane speed and the length of the final approach path.

We first observe that the ALP involves two decision problems: (1) a **sequencing** problem (which determines the sequence of plane landings) and (2) a **scheduling** decision problem (which determines the precise landing times for each plane in the sequence, subject to the separation constraints).

The ALP has the following notations:

N : the number of planes.

P : Set of N planes, $P=\{1,2,\ldots,N\}$.

R : the number of landing runways (here we take R=1).

E_i : the earliest landing time for plane $i \in P$.

L_i : the latest landing time for plane $i \in P$.

T_i : the target (preferred) landing time for plane $i \in P$.

S_{ij} : the required separation time (≥ 0) between plane i landing and plane j landing (where plane i lands before plane j), $i,j \in P$, $i \neq j$.

g_i : the penalty cost (≥ 0) per unit of time for landing before the target time T_i for plane $i \in P$.

h_i : the penalty cost (≥ 0) per unit of time for landing after the target time T_i for plane $i \in P$.

The variables are:

t_i : the actual landing time for plane $i \in P$.

α_i : how soon plane $i \in P$ lands before T_i, mathematically, the earliness $\alpha_i = \max\{0, T_i - t_i\}$.

β_i : how soon plane $i \in P$ lands after T_i, mathematically, the tardiness $\beta_i = \max\{0, t_i - T_i\}$.

$$\delta_{ij} = \begin{cases} 1 & \text{if plane } i \text{ lands before plane } j\ \forall i,j \in P,\ i \neq j. \\ 0 & \text{otherwise.} \end{cases}$$

Without significant loss of generality, we shall henceforth assume that the times E_i, L_i, and S_{ij} are integers.

3.3 Single Runway Formulation of ALP

In this section, we present an initial mixed-integer zero–one formulation of the static single runway ALP.

3.3.1 ALP Constraints

The first set of constraints are

$$E_i \leq t_i \leq L_i,\ \forall i \in P, \qquad \ldots(3.1)$$

which ensure that each plane lands within its time window. Now, considering pairs (i,j) of planes we have that

$$\delta_{ij} + \delta_{ji} = 1,\ \forall i,j \in P,\ i < j, \qquad \ldots(3.2)$$

We need to define three sets:

U : the set of pairs (i,j) of planes for which we are uncertain whether plane i lands before plane j.

V : the set of pairs (i,j) of planes for which i definitely lands before j (but for which the separation constraint is not automatically satisfied).

W : the set of pairs (i,j) of planes for which i definitely lands before j (and for which the separation constraint is automatically satisfied).

Then, we can define the set W by

$$W = \{(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} \le E_j,\ \forall i,j \in P,\ i \ne j]\} \qquad \ldots(3.3)$$

In words, i must land before j ($L_i<E_j$) and the separation constraint is automatically satisfied ($L_i+S_{ij}\le E_j$).

We can define the set V by

$$V = \{(i,j) \mid L_i < E_j \text{ and } L_i + S_{ij} > E_j,\ \forall i,j \in P,\ i \ne j\} \qquad \ldots(3.4)$$

In words, i must land before j ($L_i<E_j$) but the separation constraint is not automatically satisfied ($L_i+S_{ij}>E_j$).

Some plane lands first may have overlapping time windows. Hence, we can define the set U as:

$$U=\{(i,j) \mid i,j=1,\ldots,N, i\ne j; E_j\le E_i\le L_j \text{ or } E_j\le L_i\le L_j,\ E_i\le E_j\le L_i \text{ or } E_i\le L_j\le L_i\} \ldots(3.5)$$

We need a separation constraint for pairs of planes in V, and this is

$$t_j \ge t_i + S_{ij} \quad \forall (i,j)\in V \qquad \ldots(3.6)$$

which ensures that a time S_{ij} must elapse after the landing of plane i at t_i before plane j can land at t_j.

Finally, we need constraints to relate the α_i, β_i, and t_i variables to each other.

$$T_i - t_i \le \alpha_i \le T_i - E_i, \quad i\in P, \qquad \ldots(3.7)$$

$$t_i - T_i \le \beta_i \le L_i - T_i, \quad i\in P, \qquad \ldots(3.8)$$

3.3.2 Objective Function

We now need only to setup the objective function, minimize the deviation from the target times (T_i), and this is

$$\text{minimize} \sum_{i=1}^{N} (g_i\alpha_i + h_i\beta_i) \qquad \ldots(3.9)$$

The complete formulation (model) of the single runway problem is therefore to satisfy function (3.9) subject to relations (3.1), (3.2), and (3.6)-(3.8).

Example (3.1): For N=3, lets have the following ALP information.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Formulate this ALP:

$$Z=\sum_{i=1}^{3}(g_i\alpha_i+h_i\beta_i)$$

$Z=(10*\max\{0,155-t_1\}+10*\max\{0,t_1-155\})+(10*\max\{0,258-t_2\}+$

$10*\max\{0,t_2-258\})+(30*\max\{0,98-t_3\}+10*\max\{0,t_3-98\})$

s.t.

$129 \le t_1 \le 559$, $195 \le t_2 \le 744$, $89 \le t_3 \le 510$,

$\delta_{ij}+\delta_{ji}=1$, $\forall i,j\in P$, $i<j$,

$t_j\ge t_i+S_{ij}$ $\forall (i,j)\in V$,

$155-t_1\le\max\{0,155-t_1\}\le 26$, $258-t_2\le\max\{0,258-t_2\}\le 63$, $98-t_3\le\max\{0,98-t_1\}\le 9$,

$t_1-155\le\max\{0,t_1-155\}\le 404$, $t_2-258\le\max\{0,t_2-258\}\le 486$, $t_3-98\le\max\{0,t_3-98\}\le 412$.

Calculate the objective function Z if t_i=150, 250, 100:

we have $\alpha_i=\max\{0,T_i-t_i\}$, i=1,2,3, then:

$\alpha_i=\max\{0,155-150\}=5$, $\max\{0,258-250\}=8$, $\max\{0,98-100\}=0$.

And $\beta_i=\max\{0, t_i-T_i\}$, i=1,2,3, then:

$\beta_i=\max\{0,150-155\}=0$, $\max\{0,250-258\}=0$, $\max\{0,100-98\}=2$.

$Z=(g_1\alpha_1+h_1\beta_1)+(g_2\alpha_2+h_2\beta_2)+(g_3\alpha_3+h_3\beta_3)=10\times5+10\times8+30\times2=190$.