CHAPTER THREE

AIRCRAFT LANDING PROBLEMS (ALP)

3.5 Solving ALP using Heuristic and CE Methods

We observe that if we are given the sequence in which planes land, then we can develop an algorithm for resolving the scheduling decisions.

3.5.1 Parallel Improving Technqiue

The order between aircraft (**sequencing the aircraft**) is setup according to priority rules which are based on the variables:

- **E_i**: The priority is given to the aircraft which has the sooner earliest landing time.
- **T_i** : The priority is given to the aircraft which has the earliest target landing time.
- **L_i** : The priority is given to the aircraft which has the earliest latest landing time.
- **E_i/g_i** : The priority is given to the aircraft which has the soonest earliest time.
- **L_i/h_i** : The priority is given to the aircraft which has the soonest latest time.
- **$Ti /(g_i+h_i)$**: The priority is given to the aircraft which has the soonest target.
- **$1/(g_i+h_i)$**: The priority is given to the aircraft which causes the most important advance and lateness penalty.

Example (3.5):

Call example (3.1) we have the following priority rules:

1. E_i: we have the sequence 3,1,2.
2. T_i: we have the sequence 3,1,2.

3. L_i: we have the sequence 3,1,2.
4. E_i/g_i=(12.9,19,5,2.97), we have the sequence 3,1,2.
5. L_i/h_i=(55.9,74.9,17), we have the sequence 3,1,2.
6. $T_i/(g_i+h_i)$=(7.75,12.9,1.63), we have the sequence 3,1,2.
7. $1/(g_i+h_i)$=(0.05,0.05,0.03), we have the sequence 3,1,2 or 3,2,1.

Exercise (3.3): Find the priority rules for **Exercise (3.2).**

The adjusting landing time (**scheduling aircraft**) is the most important step in the improvement algorithm; the aim is to reduce the total cost of penalty caused by all aircraft. If we increase (reduce) the aircraft landing time, we check the interval of improving the landing aircraft time during the initialization: in this algorithm (**Parallel Improving Algorithm** (PIA)), we adjust the landing time of each aircraft during its time assignment.

Algorithm (3.1): Parallel Improving Algorithm (PIA)

Let P be the list of aircraft set up according to a priority rule and O={}.

1. $t_{P1} \leftarrow T_{P1}$; $P_1 \in O$.
2. **FOR** i = 2 : N

   $t_{Pi} \leftarrow \max(T_{Pi}, \max\limits_{Pj \in O}(t_{Pj} + S_{Pi,Pj}))$

   END {FOR i}
3. **REPEAT**

   Calculate penalty Cost Z

   IF ($t_{Pi} > T_{Pi}$)

   Reduce the landing time by 1 unit of time

   ELSE { $t_{Pi} \le T_{Pi}$}

   Increase the landing time by 1 unit of time

   END {IF}

   IF (the solution is unfeasible)

   Reject the change and keep the last feasible solution.

BREAK.

END {IF}

UNTIL (there is increase of penalty cost)

Example (3.6): For N=10 which is shown in the Appendix, we have two tables (I, II), using PIA for the 1st (7) planes.

1. The 1st step is to set up an order between aircraft according to a priority rule. Suppose that the priority rule is the earliest target time. The order is as follows:

P_i	3	4	5	6	7	1	2

2. The 2nd step is to assign landing time to the 1st aircraft in the list (P_1=3) by one of the assignment heuristic, i.e.: $t_3 = T_3$=98, O={3}, then:

P_i	3	4	5	6	7	1	2	Z
t_i	98							0

3. For the 2nd aircraft in the list (P_2=4), then

$t_4 \leftarrow \max(T_4, \max_{Pj\in O}(t_3 + S_{3,4}))$=max(106,max(98+8))=106, O={3,4}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106						0

4. For the 3rd aircraft in the list (P_3=5), then

$t_5 \leftarrow \max(T_5, \max(t_3+S_{3,5},t_4+S_{4,5}))$=max(123,max(98+8,106+8))=123, O={3,4,5}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123					0

5. For the 4th aircraft in the list (P_4=6), then

t_6=max(135,max(98+8,106+8,123+8))=135,

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135				0

O={3,4,5,6}, Z=0.

6. For the 5th aircraft in the list (P_5=7), then

t_7=max(138,max(98+8,106+8,123+8,135+8))=143≠138, O={3,4,5,6,7},

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135	143			150

here we need adjusting the landing time t_7=142, then t_6=134:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	134	142			150

And continue in decreasing until we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139			150

If we continue another step we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	122	130	138			180

Since Z=180, we ignore this step and back to the last step when Z=150.

7. For the 6th aircraft in the list (P_6=1), then

T_1=max(155,max(98+15,106+15,123+15,131+15,139+15))=155,O={3,4, 5,6,7,1}, now we need no adjusting the landing time so we obtain, Z=700

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155		150

8. For the 7th aircraft in the list (P_7=2), then

T_2=max(258,max(98+15,106+15,123+15,131+15,139+15,155+15))=258, O={3,4,5,6,7,1,2}, now we need no adjusting the landing time so we obtain, Z=150:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155	258	150

Table (3.2) shows the implementation of PIA for this example.

Table (3.2): the results obtained from PIA for example (3.6).

Stage	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**Cost Z**
1	98							0
2	98	106						0
3	98	106	123					0
4	98	106	123	135				0
5	98	106	123	131	139			150
6	98	106	118	131	139	155		150
7	98	106	118	131	139	155	258	150

Exercise (3.4): Apply PIA using T_i priority:

1. Using example (3.6) for N=8, 9, 10.
2. N=5;

	P_1	P_2	P_3	P_4	P_5
E_i	129	190	84	89	100
T_i	155	250	93	98	111
L_i	305	400	143	148	161
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

3. N=5;

	P_1	P_2	P_3	P_4	P_5
E_i	146	249	95	103	120
T_i	155	258	98	106	123
L_i	164	267	101	109	126
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

In table (3.3), we demonstrate the total penalty cost (Z) results of PIA for sequencing using different priorities.

Table (3.3): Results of PIA for different N.

N	**Penalty cost according to the priority rule's**							**Best Z**	**Opt.**
	E_i	**T_i**	**L_i**	**$1/(g_i+h_i)$**	**$T_i/(g_i+h_i)$**	**E_i/g_i**	**L_i/h_i**		
10	1280	700	2360	880	880	880	1090	700	700
15	1790	1500	2210	2210	1480	1480	1770	1480	1480
20	1790	1730	5890	5000	820	880	2140	820	820
20	4890	2520	6100	4210	2520	2650	3260	2520	2520
20	6470	5420	7060	4060	3100	3250	3220	3100	3100
30	24442	24442	24442	89766	31194	31194	37956	24442	24442
44	1550	1550	1550	73536	40330	33820	43443	1550	1550
50	18790	2530	61215	215025	152270	168505	141570	2530	1950

The best results are the shaded cells. From table (3.3), the best priorities is T_i and $T_i/(g_i+h_i)$ for PIA.