CHAPTER THREE

AIRCRAFT LANDING PROBLEMS (ALP)

3.5.2 Complete Enumeration Method (CEM)

When using CEM, in sequencing stage we will try all the possible permutation of N planes which equal to N!, while in scheduling stage we will apply two methods:

- Exhaustive Search Method (ESM): in this method we try all possibilities starting from E_i ending in L_i. The total number of all possibilities for scheduling is $\prod_{i=1}^{N}(L_i - E_i + 1)$.
- PIA.

Note that the total complexity (C(N)) for sequencing and scheduling N-planes using CEM is:

$$C(N)=N!*\prod_{i=1}^{N}(L_i - E_i + 1) \qquad \ldots(3.12)$$

For $E_i=T_i=L_i$, (Z=0) $\forall i \in P$, then C(N)=N!.

Remark (3.3): In general, if :

R: the number of pairs of aircraft which are satisfy SR's.

D: the number of pairs of aircraft which are not submitted to SR's, represented by the variables δ_{ij} in matrix A.

where $R+D=C_2^N=N*(N-1)/2$, for the ALP we have 2^D sequences can be try to find the best sequence. In some ALP, N! may be larger than 2^D and vice versa. Table (3.4) shows samples of ALP examples with C(N) and 2^D for different N.

Table (3.4) samples of ALP examples with C(N) and 2^D.

N	C(N)	N!	C_2^N	R	D	2^D
8	9.689287×10^{16}	40320	28	17	11	2048*
9	2.486859×10^{20}	362880*	36	17	19	524288
10	1.892271×10^{23}	3628800*	45	23	22	4194304
15	5.084773×10^{41}	1.3077×10^{12}*	105	44	61	2.305843×10^{18}

The cells signed with (*) means is better to be used in search to find good sequence.

3.5.2.1 Complete Enumeration using ESM

In this subsection, we introduce the results of optimal penalty cost Z using the exact ESM for N=3,…,9. The results obtained after applying TWT and find SR. The TWT very important factor to reduce the calculations, since it implies to:

1. Increase the number of R (i.e. decrease the number D) for fixed N.
2. Reduce the complexity of CEM.

These two factors will reduce the CPU time of CEM to approach the optimal solution. In table (3.5), we show the influence of TWT on R, C(N=6) and CPU time for different choices of Z_{UB}.

Table (3.5): the influence of TWT on R, C(6) and CPU time for different Z_{UB}.

i	Z_{UB}	R	C(N)	NF	CPU/s
1	0	15	6!=720	1	0.2
2	480	11	3.529993×10^{12}	8	110
3	600	9	8.627828×10^{12}	18	306
4	770	7	2.362645×10^{13}	25	1071
5	1080	6	9.397747×10^{13}	48	6427

Where NF is the number of feasible solutions in specific sequencing.

Example (3.7): lets have the following ALP:

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	133	130	99
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	4	4
2	4	0	4
3	4	4	0

The general Complexity is C(3)=6*64=384.The number of SR=3, R=3 and D=0 so we have the unique sequence π=(3,2,1), then C(3) reduces to 64 possible. Then the best solutions using CEM-ESM are:

1 - 96,127,131, Z=40.
2 - 96,128,132, Z=40.
3 - 97,127,131, Z=10.
4 - 97,128,132, Z=10.

Table (3.6) shows the results of applying CEM using ESM for N=3,4,…,9, with CPU time.

Table (3.6): The results of applying CEM using ESM for N=3,4,…,9, with CPU time.

N	N!	optimal sequence	optimal schedule	C(N)	NF	CPU/s
3	6	3,1,2	98,155,258	6	1	0.06
4	24	3,4,1,2	98,106,155,258	24	1	0.06
5	120	3,4,5,1,2	98,106,123,155,258	120	1	0.07
6	720	3,4,5,6,1,2	98,106,123,135,155,258	720	1	0.2
7	5040	3,4,5,6,7,1,2	98,106,123,131,139,155,258	7.800409×10^{11}	1	3
8	40320	3,4,5,6,7,8,1,2	98,106,122,130,138,146,161,258	9.689287e$\times10^{16}$	32	1456
9	362880	3,4,5,6,7,8,9,1,2	98,106,122,130,138,146,154,258,266	2.486859×10^{20}	>136	>3hs

3.5.2.2 Complete Enumeration using PIA

In this subsection, we introduce the results of best penalty cost Z using the heuristic PIA for N=3,…,9.

Example (3.8): call example (3.7), we have the unique sequence π=(3,2,1), then the best solution using CEM-PIA is:

1 - 97,127,131, Z=10.

Exercise (3.5): Find the optimal solution for the following ALP:

1.

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	132	129	98
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0

The general Complexity is C(3)=6*27=162.

The number of SR=3, R=3 and D=0 so we have the unique sequence π=(3,2,1), then C(3) reduces to 27 possible. Then the best solutions using:

a. CEM-ESM are:

1 - 96,127,130, Z=50.
2 - 96,127,131, Z=40.
3 - 96,128,130, Z=40.
4 - 96,128,131, Z=30.
5 - 97,127,130, Z=20.
6 - 97,127,131, Z=10.
7 - 97,128,130, Z=10.
8 - 97,128,131, Z=0.
b. CEM-PIA is: 97,128,131, Z=0.
2.

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	132	129	98
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	5	5
2	5	0	5
3	5	5	0

we have the unique sequence π=(3,2,1), then C(3) reduces to 27 possible. Then the best solutions using:

a. CEM-ESM are:

1 - 96,127,132, Z=50.
2 - 97,127,132, Z=20.
b. CEM-PIA is: 97,127,132, Z=20.

The results obtained after applying TWT and SR. Table (3.7) shows the results of applying CEM using PIA for N=3,…,9, with CPU time.

Table (3.7): Applying CEM using PIA for N=3,…,9, with CPU time.

Ns	best sequence	best scheduling	**Best Z**	**NF**
3	3,1,2	98,155,258	0	1
4	3,4,1,2	98,106,155,258	0	1
5	3,4,5,1,2	98,106,123,155,258	0	1
6	3,4,5,6,1,2	98,106,123,135,155,258	0	1
7	3,4,5,6,7,1,2	98,106,123,131,139,155,258	150	1
8	3,4,5,6,7,8,1,2	98,106,122,130,138,146,161,258	420	19
9	3,4,5,6,7,8,9,1,2	98,106,122,130,138,146,154,169,258	620	129

Note the difference in CPU time for CE using ESM and PIA.