Lecture One

Mathematical Basic Concepts

7. Sequences and Series

7.1 Sequences

Definition (7.1): The **sequence** in the field $\mathbb{F}$ is a function f, whose domain is the set of non negative (or could be positive) integer, s.t. $f:\mathbb{Z}\to\mathbb{F}$, and its denoted by $S=\{S_n\}_{n=0}^{+\infty}$.

Definition (7.2): The Sequence S is **periodic** when $\exists p\in\mathbb{Z}^+$ s.t. $s_0=s_p, s_1=s_{p+1},\ldots$, the minimum p is the **period** of S.

If $\mathbb{Z}_m=\{0,1,\ldots,m-1\}$, where $m\in\mathbb{Z}^+$, then S is digital sequence. In special case, if m=2 then S is binary sequence.

7.2 Series

Definition (7.3): An infinite series is an expression of the form:

$u_1+u_2+\ldots u_k+\ldots=\sum_{k=1}^{\infty} u_k$.

Let S_n denotes the sum of the first n terms of the series s.t.

$S_n=\sum_{k=1}^{n} u_k$, and $\{S_n\}_{n=1}^{+\infty}$ is called the **sequence of partial sums**.

$S=\sum_{k=1}^{\infty} u_k$ is called the **sum** of the series.

Theorem (7.1): A geometric series $a+ar+ar^2+\ldots+ar^{k-1}+\ldots(a\neq 0)$ is converges if $|r|<1$ and the sum is $\frac{a}{1-r}=a+ar+ar^2+\ldots+ar^{k-1}+\ldots$, and diverges if $|r|\geq 1$.