Lecture One

Mathematical Basic Concepts

9. Probability Theory

Definition (9.1) An ***experiment*** is a procedure that yields one of a given set of outcomes. The individual possible outcomes are called ***simple events***. The set of all possible outcomes is called the ***sample space***.

we only considers discrete sample spaces; that is, sample spaces with only finitely many possible outcomes. Let the simple events of a sample space S be labeled $s_1, s_2, \ldots, s_n$.

Definition(9.2) A ***probability distribution*** P on S is a sequence of numbers $p_1, p_2, \ldots, p_n$ that are all non-negative and sum to 1. The number p_i is interpreted as the probability of s_i being the outcome of the experiment.

Definition (9.3) An ***event*** E is a subset of the sample space S. The probability that event E occurs, denoted P(E), is the sum of the probabilities p_i of all simple events s_i which belong to E. If $s_i \in S$, $P(\{s_i\})$ is simply denoted by $P(s_i)$.

Definition (9.4) If E is an event, the ***complementary event*** is the set of simple events not belonging to E, denoted $\bar{E}$.

Fact (9.1) Let $E \subseteq S$ be an event.

i. $0 \le P(E) \le 1$. Furthermore, $P(S) = 1$ and $P(\varphi) = 0$. (φ is the empty set).

ii. $P(\bar{E}) = 1 - P(E)$.

iii. If the outcomes in S are equally likely, then $P(E) = |E|/|S|$.

Definition (9.5) Two events E_1 and E_2 are called mutually exclusive if $P(E_1 \cap E_2)=0$. That is, the occurrence of one of the two events excludes the possibility that the other occurs.

Fact (9.2) Let E_1 and E_2 be two events:

i. If $E_1 \subseteq E_2$, then $P(E_1) \leq P(E_2)$.

ii. $P(E_1 \cup E_2)+P(E_1 \cap E_2)=P(E_1)+P(E_2)$. Hence, if E_1 and E_2 are mutually exclusive, then $P(E_1 \cup E_2)=P(E_1) + P(E_2)$.