

ex. 7  $\int (1 - \sin^2 4x) \cdot \cos 4x \, dx.$

Sol  $\int (\cos 4x - \sin^2 4x \cdot \cos 4x) \, dx$   
 $= \int \cos 4x \, dx - \int \sin^2 4x \cdot \cos 4x \, dx \times \frac{4}{4}$   
 $= \int \cos 4x \, dx \times \frac{4}{4} - \int \sin^2 4x \cdot \cos 4x \, dx \times \frac{4}{4}$   
 $= \frac{1}{4} \sin 4x - \frac{1}{4} \times \frac{\sin^3 4x}{3} + c$   
 $= \frac{1}{4} \sin 4x - \frac{1}{12} \sin^3 4x + c$

ex. 8

$\int \frac{\sin \sqrt{x}}{\sqrt{x}}$

Sol  $\int \sin \sqrt{x} \times \frac{1}{\sqrt{x}} \, dx \times \frac{2}{2}$   
 $= 2 \int \sin \sqrt{x} \times \frac{1}{2\sqrt{x}} \, dx = -2 \cos \sqrt{x} + c$

ex. 9

$\int \frac{\cot^2 \sqrt{x}}{\sqrt{x}}$

$\cot^2 \sqrt{x} = \csc^2 \sqrt{x} - 1$

$= \int \frac{\csc^2 \sqrt{x} - 1}{\sqrt{x}} \, dx = \int \frac{\csc^2 \sqrt{x}}{\sqrt{x}} \, dx - \int \frac{1}{\sqrt{x}} \, dx$

$= \int \frac{\csc^2 \sqrt{x}}{\sqrt{x}} \, dx \times \frac{2}{2} - \int x^{-\frac{1}{2}} \, dx$

$= 2 \int \csc^2 \sqrt{x} \times \frac{dx}{2\sqrt{x}} - \int x^{-\frac{1}{2}} \, dx$

$= 2 \times -\cot \sqrt{x} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= -2 \cot \sqrt{x} - 2x^{\frac{1}{2}} + c$

$= -2 \cot \sqrt{x} - 2\sqrt{x} + c$

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ملحوظة  
لايجاد التفاضل للدوال التالية نطبق القواعد التالية

$$\textcircled{1} \int \tan x dx = -\ln |\cos x| + c$$

$$\textcircled{2} \int \cot x dx = \ln |\sin x| + c$$

$$\textcircled{3} \int \sec x dx = \ln |\sec x + \tan x| + c$$

$$\textcircled{4} \int \csc x dx = -\ln |\csc x + \cot x| + c$$

ex. 1  
sol

$$\int \tan(3x+5) dx$$

$$\int \tan(3x+5) dx \neq \frac{3}{3}$$

$$= \frac{1}{3} \int \tan(3x+5) \cdot 3 dx = -\ln |\cos(3x+5)|$$

ex. 2  
u

$$\int \frac{\cos x}{1 - \sin^2 x} dx$$

sol

$$\int \frac{\cos x}{\cos^2 x} dx = \int \frac{1}{\cos x} dx = \int \sec x dx$$

$$= \ln |\sec x + \tan x| + c$$

ex. 3  
sol

$$\int \frac{\sin x + \cos x}{\cos x} dx$$

$$\int \frac{\sin x}{\cos x} dx + \int \frac{\cos x}{\cos x} dx = \int \tan x dx + \int dx$$

$$= -\ln |\cos x| + x + c$$

✓



✓



## Integration of The $e^x$ and $a^x$

\*  $x$  جلا تامل دالة  $(e^x)$  و  $(a^x)$  طبق التالي

$$\textcircled{1} \int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C$$

أي تنزلها نفسها عند توصل مشتقة الأس

$$\textcircled{2} \int a^{f(x)} \cdot f'(x) dx = \frac{a^{f(x)}}{\ln a}$$

عند توصل مشتقة الدالة

ex 1  $\int e^x dx$

Sol  $= e^x + C$

ex. 2  $\int e^{3x} dx$

Sol  $\int e^{3x} dx \cdot \frac{3}{3} = \frac{1}{3} \int e \cdot 3 dx$   
 $= \frac{1}{3} e^{3x} + C$

ex-3  $\int e^{\sin x} \cdot \cos x dx$

Sol  $= e^{\sin x} + C$

A



$$\text{ex 4)} \int 2^x dx$$

$$\underline{\text{sol}} = \frac{2^x}{\ln 2} + c$$

$$\text{ex 5)} \int x \cdot 3^{x^2} dx$$

$$= \int x \cdot 3^{x^2} dx \times \frac{2}{2}$$

$$= \frac{1}{2} \int 3^{x^2} \cdot 2x dx = \frac{1}{2} \times \frac{3^{x^2}}{\ln 3} + c$$

$$\text{ex. 6)} \int e^{\ln 5^x} dx$$

$$\underline{\text{sol}} \int e^{\ln 5^x} dx = \int 5^x dx$$

$$= \frac{5^x}{\ln 5} + c$$

$$\text{ex. 7)} \int 3^{\sec x} \cdot \sec x \cdot \tan x dx$$

$$= \frac{3^{\sec x}}{\ln 3} + c$$

$$\text{ex. 8)} \int \frac{\ln^2 x}{x} dx$$

$$\underline{\text{sol}} \int \ln^2 x \times \frac{1}{x} dx = \frac{\ln^3 x}{3} + c$$



ex. 9

$$\int \frac{dx}{x \ln x}$$

Sol

$$\int \frac{dx}{\ln x} \cdot \frac{1}{x} = \ln(\ln x) + c$$

ex. 10

$$\int \frac{\cot x}{\csc x} dx$$

$$\cot = \frac{\cos x}{\sin x}$$

$$\csc = \frac{1}{\sin x}$$

Sol

$$\int \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \int \frac{\cos x}{\sin x} \cdot \sin x dx$$

$$= \int \cos x dx = \sin x + c$$

ex. 11

$$\int \frac{\sec^3 x + e^{\sin x}}{\sec x} dx$$

$$\sec = \frac{1}{\cos x}$$

Sol

$$\int \frac{\sec^3 x}{\sec x} dx + \int \frac{e^{\sin x}}{\sec x} dx$$

$$= \int \sec^2 x dx + \int \frac{e^{\sin x}}{\frac{1}{\cos x}} dx$$

$$= \int \sec^2 x dx + \int e^{\sin x} \cdot \cos x dx$$

$$= \tan x + e^{\sin x} + c$$

1.

