(ex.4) $\int e^x \cdot \cos x \, dx$

Sol let $u = e^x \Rightarrow du = e^x dx$

$dv = \cos x dx \Rightarrow v = \sin x$

$\int u \, dv = uv - \int v \, du$.

$\therefore \int e^x \cdot \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$

let $u = e^x \Rightarrow du = e^x dx$

$dv = \sin x dx \Rightarrow v = -\cos x$.

$= e^x \cdot \sin x - [e^x(-\cos x) - \int -\cos x \, e^x dx]$

$= e^x \cdot \sin x + e^x \cos x - \int e^x \cdot \cos x \, dx$

$\therefore \int e^x \cdot \cos x \, dx = e^x \cdot \sin x + e^x \cos x - \int e^x \cdot \cos x \, dx$

$2 \int e^x \cdot \cos x \, dx = e^x \cdot \sin x + e^x \cdot \cos x$

$\therefore \int e^x \cdot \cos x \, dx = \frac{1}{2} [e^x \cdot \sin x + e^x \cdot \cos x] + c$

ex.5

$\int x \cdot \ln x \, dx$

Sol

let $u = \ln x$ $\Rightarrow du = \frac{1}{x} dx$, $dv = x\,dx$ $\Rightarrow v = \frac{1}{2}x^2$

Sol

$\int u\,dv = uv - \int v\,du$.

$= \int x \cdot \ln x \, dx = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2}\int x\,dx =$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2} * \frac{x^2}{2} + c$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{4} \cdot x^2 + c$

ex.6

$\int \frac{x}{\sqrt{x-1}} dx$

Sol

let u $\Rightarrow du = dx$

$dv = \frac{dx}{\sqrt{x-1}}$ $\Rightarrow v = \int \frac{dx}{\sqrt{x-1}} = \int (x-1)^{-\frac{1}{2}} dx$

$= 2(x-1)^{\frac{1}{2}}$

$\therefore uv - \int v\,du$

$= x \cdot 2\sqrt{x-1} - \int 2(x-1)^{\frac{1}{2}} dx = 2x\sqrt{x-1} - 2\int (x-1)^{\frac{1}{2}} dx$

$= 2x\sqrt{x-1} - 2 * \frac{(x-1)^{3/2}}{\frac{3}{2}} + c$

$= 2x\sqrt{x-1} - 2 * \frac{2}{3}\sqrt{(x-1)^3} + c$

(2) Tabular integration التكامل بالجدول

* وهو جزء من طريقة التكامل بالتجزئة وتستخدم هذه الطريقة عندما توجد دالتان مقرونتان احدهما قابلة للتكامل والأخرى قابلة للاشتقاق الى الصفر.

ex 1 $\int_0^1 x^2 \cdot e^x \, dx$

derivative المشتقة		integration التكامل
x^2	+	e^x
$2x$	−	e^x
2	+	e^x
0		e^x

$$\Rightarrow \int_0^1 x^2 \cdot e^x \, dx = x^2 e^x - 2xe^x + 2e^x \Big|_0^1$$

$$= [(1)^2 \cdot e^1 - 2(1)e^1 + 2e^1] - [0e^0 - 2 \cdot 0 \cdot e^0 + 2e^0]$$

$$= e^1 - 2$$

ex.2 $\int x^3 \sin x \, dx$

derivative		integrals
x^3	+	$\sin x$
$3x^2$	−	$-\cos x$
$6x$	+	$-\sin x$
6	−	$\cos x$
0		$\sin x$

$$\therefore \int x^3 \sin x \, dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$$

(ex.3) $\int (x^2+x+1)\cdot e^x \, dx$

Sol

derivation		integration
x^2+x+1	+	e^x
$2x+1$	−	e^x
2	+	e^x
0		e^x

$\therefore \int (x^2+x+1)\cdot e^x \, dx = (x^2+x+1)\cdot e^x - (2x+1)\cdot e^x + 2e^x + c$

$= x^2e^x + xe^x + e^x - 2xe^x - e^x + 2e^x + c$

$= x^2e^x - xe^x + 2e^x + c$

(ex.4)

$\int x^3 \cdot e^x \, dx$

derivation		integral
x^3	+	e^x
$3x^2$	−	e^x
$6x$	+	e^x
6	−	e^x
0		e^x

$\therefore \int x^3 \cdot e^x \, dx = x^3 \cdot e^x - 3x^2 \cdot e^x + 6xe^x - 6e^x + c$

ex.5 $\int x^3 \cdot 2^x \, dx$

Sol

derivative		integral
x^3	+	2^x
$3x^2$	−	$\frac{2^x}{\ln 2}$
$6x$	+	$\frac{2^x}{(\ln 2)^2}$
6	−	$\frac{2^x}{(\ln 2)^3}$
0		$\frac{2^x}{(\ln 2)^4}$

$$\int x^3 \cdot 2^x \, dx = x^3 \cdot \frac{2^x}{\ln 2} - 3x^2 \cdot \frac{2^x}{(\ln 2)^2} + 6x \cdot \frac{2^x}{(\ln 2)^3} - 6 \frac{2^x}{(\ln 2)^4} + C$$

ex.6 $\int x^2 \cdot e^{x^2} \, dx$

Sol

Let $u = x^2$ $\quad dv = e^{x^2}$

$du = 2x \, dx$ $\quad v = \int \frac{1}{2} e^{x^2}$

$uv - \int v \, du$