

ex. 4 $\int e^x \cdot \cos x \, dx$

Sol let $u = e^x \Rightarrow du = e^x dx$
 $dv = \cos x dx \Rightarrow v = \sin x$

$$\int u \, dv = uv - \int v \, du.$$

$$\therefore \int e^x \cdot \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx$$

let $u = e^x \Rightarrow du = e^x dx$
 $dv = \sin x dx \Rightarrow v = -\cos x.$

$$= e^x \sin x - [e^x (-\cos x) - \int -\cos x e^x dx]$$

$$= e^x \sin x + e^x \cos x - \int e^x \cdot \cos x \, dx$$

$$\therefore \int e^x \cdot \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cdot \cos x \, dx$$

$$\Rightarrow 2 \int e^x \cdot \cos x \, dx = e^x \sin x + e^x \cos x$$

$$\therefore \int e^x \cdot \cos x \, dx = \frac{1}{2} [e^x \sin x + e^x \cos x] + c$$



ex. 5

$$\int x \cdot \ln x \, dx$$

Sol

let $u = \ln x$
↓
 $du = \frac{1}{x} dx$

$$dv = x \, dx$$

↓
 $v = \frac{1}{2} x^2$

Sol

$$\int u \, dv = uv - \int v \, du$$

$$= \int x \cdot \ln x \, dx = \frac{1}{2} x^2 \cdot \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} \, dx$$

$$= \frac{1}{2} x^2 \cdot \ln x - \frac{1}{2} \int x \, dx =$$

$$= \frac{1}{2} x^2 \cdot \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$= \frac{1}{2} x^2 \cdot \ln x - \frac{1}{4} \cdot x^2 + c$$

ex. 6

$$\int \frac{x}{\sqrt{x-1}} \, dx$$

Sol

let u
↓
 $du = dx$

$$dv = \frac{dx}{\sqrt{x-1}}$$

↓
 $v = \int \frac{dx}{\sqrt{x-1}} = \int (x-1)^{-\frac{1}{2}} dx$

$$= 2(x-1)^{\frac{1}{2}}$$

$$uv - \int v \, du$$

$$\therefore = x \cdot 2\sqrt{x-1} - \int 2(x-1)^{\frac{1}{2}} dx = 2x\sqrt{x-1} - 2 \int (x-1)^{\frac{1}{2}} dx$$

$$= 2x\sqrt{x-1} - 2 \cdot \frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

IV



$$= 2x\sqrt{x-1} - 2 \cdot \frac{2}{3} \sqrt{(x-1)^3} + c$$

(2) التكامل بالجداول Tabular integration

* وهو جزء من طريقة التكامل بالتجزئة وتستخدم هذه الطريقة عندما توجد دالتان طرفيتان أحدهما قابلة للتكامل والآخرى قابلة للإستتقاك إلى الصفر.

ex 1 $\int_0^1 x^2 \cdot e^x dx$

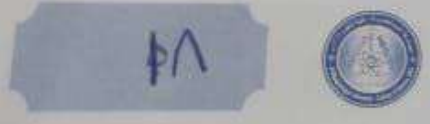
derivative المشتق		integration التكامل
x^2	+	e^x
$2x$	-	Δe^x
2	+	Δe^x
0		Δe^x

$$\begin{aligned} \Rightarrow \int_0^1 x^2 \cdot e^x dx &= x^2 e^x - 2x e^x + 2e^x \Big|_0^1 \\ &= [(1)^2 \cdot e^1 - 2(1)e^1 + 2e^1] - [0e^0 - 2 \cdot 0 \cdot e^0 + 2e^0] \\ &= e^1 - 2 \end{aligned}$$

ex. 2 $\int x^3 \sin x dx$

derivative		integrals
x^3	+	$\sin x$
$3x^2$	-	$\Delta \cos x$
$6x$	+	$\Delta -\sin x$
6	-	$\Delta \cos x$
0		$\Delta \sin x$

$$\int_0^0 x^3 \sin x dx = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$$



✍

ex. 3) $\int (x^2 + x + 1) \cdot e^x dx$

Sol

derivation

integral

$$\begin{array}{r} x^2 + x + 1 \\ 2x + 1 \\ 2 \\ 0 \end{array} \begin{array}{l} + \\ - \\ + \\ - \end{array} \begin{array}{l} e^x \\ \Delta e^x \\ \Delta e^x \\ \Delta e^x \end{array}$$

$$\begin{aligned} \int (x^2 + x + 1) \cdot e^x dx &= (x^2 + x + 1) \cdot e^x - (2x + 1) \cdot e^x + 2e^x + c \\ &= x^2 e^x + x e^x + e^x - 2x e^x - e^x + 2e^x + c \\ &= x^2 e^x - x e^x + 2e^x + c \end{aligned}$$

ex. 4

$$\int x^3 \cdot e^x dx$$

derivation

integral

$$\begin{array}{r} x^3 \\ 3x^2 \\ 6x \\ 6 \\ 0 \end{array} \begin{array}{l} + \\ - \\ + \\ - \\ + \end{array} \begin{array}{l} e^x \\ \Delta e^x \\ \Delta e^x \\ \Delta e^x \\ \Delta e^x \end{array}$$

$$\int x^3 \cdot e^x dx = x^3 \cdot e^x - 3x^2 \cdot e^x + 6x e^x - 6e^x + c$$



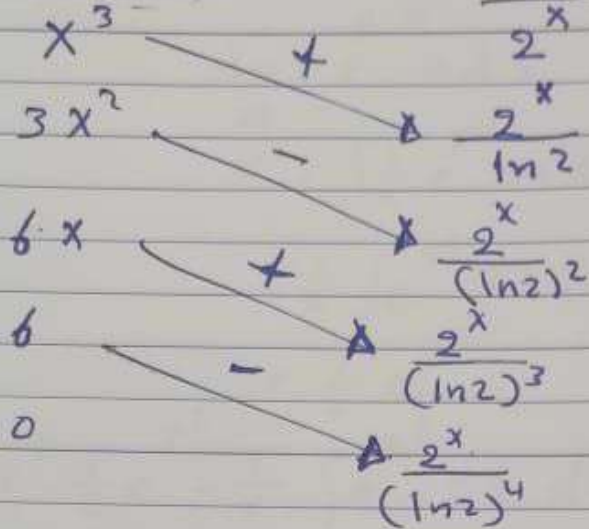
ex. 5

$$\int x^3 \cdot 2^x dx$$

Sol

derivativa

Integral.



$$\int x^3 \cdot 2^x dx = x^3 \cdot \frac{2^x}{\ln 2} - 3x^2 \cdot \frac{2^x}{(\ln 2)^2} + 6x \cdot \frac{2^x}{(\ln 2)^3} - 6 \frac{2^x}{(\ln 2)^4} + C$$

ex. 6

$$\int x^2 \cdot e^{x^2} dx$$

Sol

Let

$$u = x^2$$

$$du = 2x dx$$

$$dv = e^{x^2}$$

$$v = \frac{1}{2} e^{x^2}$$

$$uv - \int v du$$

aplikasikan



6.

