ex.7 $\int x^3 \cdot e^{x^2}\, dx$

Sol

$$\int x^2 \cdot e^{x^2} \cdot x\, dx$$

let $u = x^2$ $\Downarrow$ $du = 2x\,dx$

$dv = e^{x^2} \cdot x\,dx \ast \frac{2}{2}$

$\Downarrow$

$V = \int \frac{1}{2} e^{x^2} \cdot 2x\,dx$

$= \frac{1}{2} \int e^{x^2} \cdot 2x\,dx$

$= \frac{1}{2} e^{x^2}$

$$\therefore \int x^3 \cdot e^{x^2}\, dx = x^2 \cdot \frac{1}{2} e^{x^2} - \frac{1}{2} \int e^{x^2} \cdot 2x\, dx$$

$$= x^2 \cdot \frac{1}{2} e^{x^2} - \frac{1}{2} e^{x^2} + c$$

Odd and even powers of sine and Cosine

* في حالة إعطاء (sin) او (cos) مرفوع الى إس زوجي او فردي فإننا نتبع مايلي .

① اذا كان إس (sin) او (cos) فردي فإننا نطبق العلاقة :

$$\boxed{\cos^2 x = 1-\sin^2 x} \quad \text{أو} \quad \boxed{\sin^2 x = 1-\cos^2 x}$$

ex.1

$$\int \cos^5 5x \, dx$$

∵ الأس فردي

Sol

$$\int (\cos^2 5x)^2 \cdot \cos 5x \, dx = \int (1-\sin^2 5x)^2 \cdot \cos 5x \, dx$$

$$= \int (1-2\sin^2 5x + \sin^4 5x) \cdot \cos 5x \, dx$$

$$= \int \cos 5x \, dx \, \frac{5}{5} - 2\int \sin^2 5x \cdot \cos 5x * \frac{5}{5} +$$

$$\int \sin^4 5x \cdot \cos 5x \, dx * \frac{5}{5}$$

$$= \frac{1}{5}\int \cos 5x \cdot 5 \, dx - \frac{2}{5}\int \sin^2 5x \, 5 \cos 5x \, dx$$

$$+ \frac{1}{5}\int \sin^4 5x \cdot 5\cos 5x \, dx$$

$$= \frac{1}{5}\sin 5x - \frac{2}{5} * \frac{1}{3}\sin^3 5x + \frac{1}{5} * \frac{1}{5}\sin^5 5x + c$$

$$= \frac{1}{5}\sin 5x - \frac{2}{15}\sin^3 5x + \frac{1}{25}\sin^5 5x + c$$

ex.2 $\int \sin^3 x \, dx$

Sol $\int \sin^2 x \sin x \, dx = \int (1-\cos^2 x) \sin x \, dx$

$= \int \sin x \, dx - \int \cos^2 x \cdot \sin x \, dx \ast \frac{-1}{-1}$

$= \int \sin x \, dx + \int \cos^2 x \cdot -\sin x \, dx$

$= -\cos x + \frac{1}{3} \cos^3 x + c$

ب) إذا كان أس (sin) أو (cos) زوجي فإننا نطبق قانون ضعف الزاوية ،

$\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$ أو $\boxed{\cos^2 x = \frac{1}{2}(1+\cos 2x)}$

منه ،

ex.1 $\int \cos^2 2x \, dx$

Sol

$\int \frac{1}{2}(1+\cos 4x) \, dx = \frac{1}{2} \int (1+\cos 4x) \, dx$

$= \frac{1}{2} \left[\int dx + \int \cos 4x \, dx \ast \frac{4}{4} \right]$

$= \frac{1}{2} \left[x + \frac{1}{4} \sin 4x \right] + c$

$= \frac{1}{2} x + \frac{1}{8} \sin 4x + c$

ex.2 $\int \sin^4 3\theta \, d\theta$

Sol $\int (\sin^2 3\theta)^2 \, d\theta = \int \left[\frac{1}{2}(1-\cos 6\theta)\right]^2 d\theta$

$= \frac{1}{4}\int (1-\cos 6\theta)^2 d\theta = \frac{1}{4}\int (1 - 2\cos 6\theta + \cos^2 6\theta) d\theta$

$= \frac{1}{4}\left[\int d\theta - 2\int \cos 6\theta \, d\theta + \underline{\int \cos^2 6\theta \, d\theta}\right]$

$= \frac{1}{4}\left[\int d\theta - 2\int \cos 6\theta \, d\theta * \frac{6}{6} + \frac{1}{2}\int (1+\cos 12\theta) \, d\theta\right]$

$= \frac{1}{4}\left[\int d\theta - \frac{2}{6}\int \cos 6\theta \cdot 6 \, d\theta + \frac{1}{2}\int d\theta\right.$
$\left. + \frac{1}{2}\int \cos 12\theta \, d\theta * \frac{12}{12}\right.$

$= \frac{1}{4}\left[\int d\theta - \frac{1}{3}\int \cos 6\theta \cdot 6 \, d\theta + \frac{1}{2}\int d\theta + \right.$
$\left. \frac{1}{24}\int \cos 12\theta \cdot 12 \, d\theta\right.$

$= \frac{1}{4}\left[\theta - \frac{1}{3}\sin 6\theta + \frac{1}{2}\theta + \frac{1}{24}\sin 12\theta\right] + c$

ندخل $\frac{1}{4}$ على القوس

$= \frac{1}{4}\theta - \frac{1}{12}\sin 6\theta + \frac{1}{8}\theta + \frac{1}{96}\sin 12\theta + c$

$= \frac{3}{8}\theta - \frac{1}{12}\sin 6\theta + \frac{1}{96}\sin 12\theta + c$

* procedure of sin and cos

* في حالة وجود (sin) او (cos) مضروبان مع بعضهما وكلاهما مرفوع الى أس وكالتالي

$$\int \sin^m x \cdot \cos^n x \, dx$$

فهناك ثلاث حالات للحل :-

① اذا كان (M) فردي و (n) زوجي فنطبق العلاقة

$$\sin^2 x = 1 - \cos^2 x$$

ex ① $\int \sin^5 x \cdot \cos^2 x \, dx$

m = فردي ، n = فردي

Sol

$$\int (\sin^2 x)^2 \cdot \sin x \cdot \cos^2 x \, dx$$

$$= \int (1 - \cos^2 x)^2 \cdot \sin x \cdot \cos^2 x \, dx$$

$$= (1 - 2\cos^2 x + \cos^4 x) \cdot \sin x \cdot \cos^2 x \, dx$$

$$= \int \sin x \cdot \cos^2 x * \frac{-1}{-1} dx - 2 \int \sin x \cdot \cos^4 x \cdot \frac{-1}{-1} dx$$

$$+ \int \cos^6 x \cdot \sin x * \frac{-1}{-1}$$

$$= -\frac{\cos^3 x}{3} + 2\frac{\cos^5 x}{5} - \frac{\cos^7 x}{7} + C$$