ناقش

- Discuss The continuity of the function at $x=0$, at $x=1$

س ناقش تقاربية الدالة عند . $x=0$ ، $x=1$

$$f(x)=\begin{cases} x+\frac{1}{x}, & x<0 \\ -x^3 & 0\le x<1 \\ -1 & 1\le x<2 \end{cases}$$

**Sol** at $\boxed{x=0}$

① $f(x) = -x^3 = -(0)^3 = 0$

② $\lim\limits_{x \to 0^+} (-x^3) = -(0)^3 = 0$

$\lim\limits_{x \to 0^-} \left(x+\frac{1}{x}\right) = 0 + \frac{1}{0} = \infty$

$\therefore \lim\limits_{x \to 0^+} f(x) \neq \lim\limits_{x \to 0^-} f(x)$

2) at $\boxed{x=1}$

① $f(x) = -1$

$f(1) = -1$

② $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1} (-1) = -1$

$\lim\limits_{x \to 1^-} (-x^3) = -(1)^3 = -1$

$\therefore \lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^-} f(x) = f(1) = -1$

$\therefore$ The function is Continuos at $\boxed{x=1}$

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find $k$ that make $f(x)$ cont at $x=2$

$$f(x)=\begin{cases} kx^2 & x\le 2 \\ k+2x & x>2 \end{cases}$$

Sol ① $\lim\limits_{x\to 2^+}(k+2x) = \lim\limits_{x\to 2^-} kx^2$

$k+4 = k(4) = 4k$

$\Rightarrow k = \frac{4}{3}$

② $f(2) = 4k = \frac{16}{3}$

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# Infinity as a limit

في هذا النوع من الغايات عندما تكون الدالة مقتربة من
∞ فإننا ننظر الى اعلى أس لـ (x) ونقسم عليه جميع حدود
الدالة مع الانتباه الى الكميات غير المعرفة

**ex. 1** $\lim\limits_{x \to \infty} \dfrac{5x^2-3}{3x+1}$

**Sol:** $\lim\limits_{x \to \infty} \dfrac{5x^2-3}{3x+1} = \dfrac{\frac{5x^2}{x^2} - \frac{3}{x^2}}{\frac{3x}{x^2} + \frac{1}{x^2}} = \dfrac{5 - \frac{3}{\infty}}{\frac{3}{\infty} + \frac{1}{\infty}}$

$= \dfrac{5-0}{0+0} = \dfrac{5}{0} = \infty$

**ex 2** $\lim\limits_{x \to \infty} \dfrac{5x^3+2x^2+1}{x^3+3x}$

**sol:** $\lim\limits_{x \to \infty} \dfrac{\frac{5x^3}{x^3} + \frac{2x^2}{x^3} + \frac{1}{x^3}}{\frac{x^3}{x^3} + \frac{3x}{x^3}} = \dfrac{5 + \frac{2}{\infty} + \frac{1}{\infty}}{1 + \frac{3}{\infty}}$

$= \dfrac{5+0+0}{1+0} = \dfrac{5}{1} = 5$

**Ex 3** $\lim\limits_{x \to \infty} \dfrac{8x^2+7x}{4x^2}$

**Sol:** $\lim\limits_{x \to \infty} \dfrac{\frac{8x^2}{x^2} + \frac{7x}{x^2}}{\frac{4x^2}{x^2}} = \dfrac{8 + \frac{7}{\infty}}{4} = \dfrac{8+0}{4} = 2$

**ex ④** $$\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}} = \frac{\sqrt{\frac{x}{x}}}{\sqrt{\frac{x}{x}+\sqrt{\frac{x}{x^2}+\sqrt{\frac{x}{x^4}}}}}$$

$$= \frac{\sqrt{1}}{\sqrt{1+\sqrt{\frac{1}{\infty}}+\sqrt{\frac{1}{\infty}}}} = \frac{\sqrt{1}}{\sqrt{1+\sqrt{0}+\sqrt{0}}} = 1$$

**ex ⑤** $$\lim_{x\to\infty} \frac{(x^2+2x+1)^{50}}{(x^2+1)^{100}} = \_\_\_\_$$

**Sol** $$\lim_{x\to\infty} \frac{(x^2+2x+1)^{50}}{(x^2+1)^{100}} = \lim_{x\to\infty} \frac{\left[(x+1)^2\right]^{50}}{(x^2+1)^{100}}$$

اخذنا مربع كامل للعدد

$$= \lim_{x\to\infty} \frac{(x+1)^{100}}{(x^2+1)^{100}}$$

$$= \lim_{x\to\infty} \left[\frac{x+1}{x^2+1}\right]^{100} = \left[\frac{\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}\right]^{100}$$

$$= \left[\frac{\frac{1}{\infty}+\frac{1}{\infty}}{1+\frac{1}{\infty}}\right]^{100} = \left[\frac{\frac{1}{0}+\frac{1}{0}}{1+\frac{1}{0}}\right]^{100} = \left[\frac{0}{1}\right] = 0$$

ملاحظة: عند وجود الصيغة التالية (رقم أو متغير $\mp \sqrt{\text{دالة}}$) أو بالعكس

وكانت الغاية تقترب من $\infty$ فإننا نضرب بالعامل المرافق

**ex 6** $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$

**Sol** $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right) * \dfrac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}$

$$= \lim_{x\to\infty}\frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}} = \frac{-\frac{x}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}} = \frac{-1}{1+\sqrt{1+\frac{1}{\infty}}}$$

$$= -\frac{1}{1+\sqrt{1}} = -\frac{1}{2}$$

**ex 7** $\lim_{x\to\infty}\left(x\sqrt{x^2+1}-x^2\right)$

**Sol.** $\lim_{x\to\infty}\left(x\sqrt{x^2+1}-x^2\right) * \dfrac{x\sqrt{x^2+1}+x^2}{x\sqrt{x^2+1}+x^2}$

$$= \lim_{x\to\infty}\frac{x^2(x^2+1)-x^4}{x\left(\sqrt{x^2+1}+x^2\right)} = \lim_{x\to\infty}\frac{\cancel{x^4}+x^2-\cancel{x^4}}{x\left(\sqrt{x^2+1}+x^2\right)}$$

نختصر x كعامل مشترك

عامل مشترك

$$\lim_{x\to\infty}\frac{\cancel{x^2}}{\cancel{x}\left(\sqrt{x^2+1}+x\right)} = \lim_{x\to\infty}\frac{x}{\sqrt{x^2+1}+x}$$

$$= \frac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}+\frac{x}{x}} = \frac{1}{\sqrt{1+\frac{1}{\infty}}+1} = \frac{1}{\sqrt{1}+1}$$

$$= \frac{1}{2}$$