Inequalities – (المتراجحات)

Def If $a, b \in R$ then

① $a < b \Rightarrow b-a$ is positive or $b-a>0$

② $a \leq b \Rightarrow a<b$ or $a=b$

(properties of inequalities) خواص المتراجحات

let a, b, c and d be real numbers.

① If $a<b$ and $b<c$, then $a<c$

② If $a<b$ then $a+c<b+c$ and $a-c<b-c$

③ If $a<b$ and $c>0$, then $ac<bc$

④ If $a<b$ and $c<0$ then $ac>bc$

⑤ If $a<b$ and $c<d$ then $a+c<b+d$.

⑥ If a and b are both positive or both negative (موجبة)

then $\frac{1}{a} > \frac{1}{b}$

⑦ If $a>0$, $b>0$ and $a<b$ then $a^2<b^2$

س حل المتراجحات التالية و ارسمها على خط الاعداد

① $2x-3<7$

② $3+7x\leq 2x-9$

③ $7\leq 2-5x<9$

④ $2x-1\leq 2-x<3x+10$

⑤ $x^2-3x>10$

⑥ $\frac{2x-5}{x-2}\leq 1$

⑦ $\frac{1}{x-2}>\frac{2}{x+3}$

⑧ $\frac{x-2}{x+1}\leq 1$

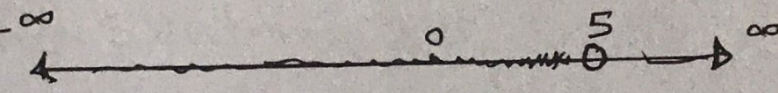
⑨ $(x-2)^2\geq 0$

Solution — الحل —

① $2x-3<7 \xrightarrow[\text{لطرفي المتراجحة}]{\text{اضافة } 3} 2x<10 \xrightarrow{\text{نضرب } \frac{1}{2}}$

$x<5$

∴ The solution is $\{x: x<5\}=(-\infty, 5)$

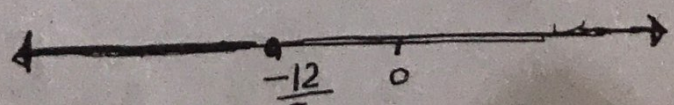


② $3+7x\leq 2x-9 \xrightarrow[\text{لطرفي المتراجحة}]{\text{اضافة } -3} 7x\leq 2x-12$

$\xrightarrow[\text{لطرفي المتراجحة}]{\text{اضافة } (-2x)} 5x\leq -12 \xrightarrow{\text{نضرب } \frac{1}{5}} x\leq -\frac{12}{5}$

∴ The solution Set is :

$\{x: x\leq -\frac{12}{5}\}=(-\infty, -\frac{12}{5}]$

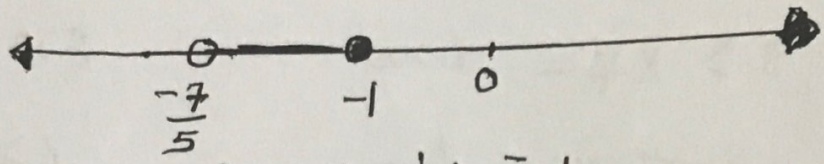


(9)

③ $7 \leq 2-5x < 9 \xrightarrow{\text{اضف } -2} 5 \leq -5x < 7$

$\xrightarrow{\text{نضرب } -\frac{1}{5}} -1 \geq x > -\frac{7}{5} \Rightarrow -\frac{7}{5} < x \leq -1$

∴ The solution set is $\{x : -\frac{7}{5} < x \leq -1\}$



طريقة اخرى للحل هي

$7 \leq 2-5x < 9 \Rightarrow 7 \leq 2-5x$ and $2-5x$

$\Downarrow$

$\Rightarrow 5 \leq -5x$ and $-5x$

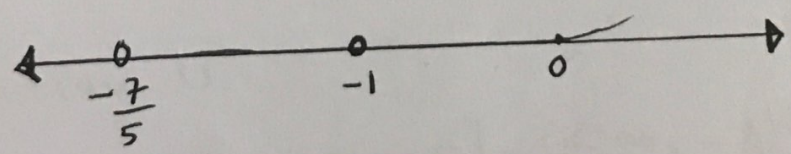
$-1 \geq x$ and $x > -\frac{7}{5}$

$\Rightarrow x \leq -1$ and $x > -\frac{7}{5}$

∴ The solution set is

$\{x : x \leq -1\} \cap \{x : x > -\frac{7}{5}\}$

$= (-\infty, -1] \cap (-\frac{7}{5}, \infty) = (-\frac{7}{5}, -1]$



④ $2x-1 \leq 2-x < 3x+10$

$\Rightarrow 2x-1 \leq 2-x$ and $2-x < 3x+10$

$\Rightarrow 2x \leq 3-x$ and $-x < 3x+8$

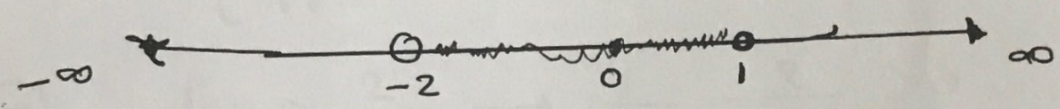
$\rightarrow 3x \leq 3$ and $-4x < 8$

$\rightarrow x \leq 1$ and $x > -2$

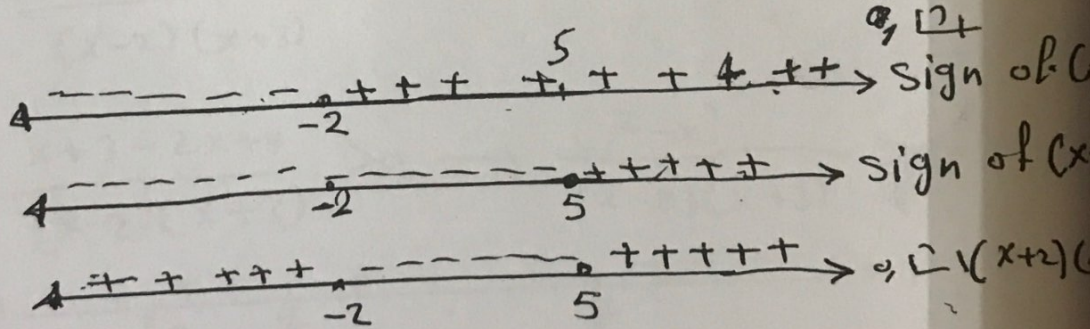
∴ The solution set is

$\{x : x \leq 1\} \cap \{x : x > -2\}$

$= (-\infty, 1] \cap (-2, \infty) = (-2, 1]$

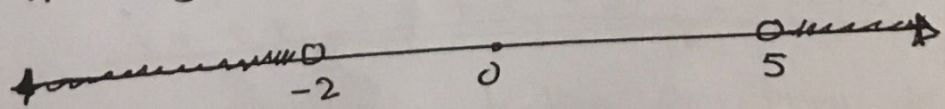


⑤ $x^2 - 3x > 10 \rightarrow x^2 - 3x - 10 > 0 \Rightarrow (x+2)(x-5)$



The solution is

$[x : x < -2\} \cup \{x ! x > 5] = (-\infty, -2) \cup (5, \infty)$

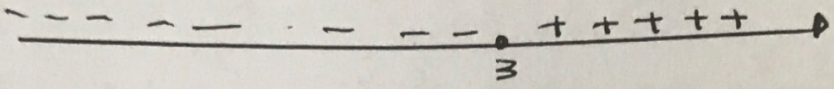


(11)

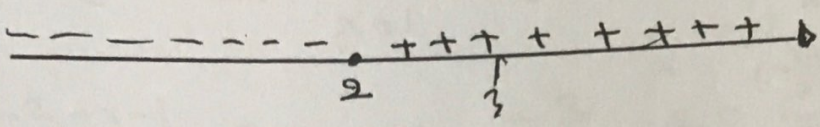
⑥ $\frac{2x-5}{x-2} \leq 1 \xrightarrow[\text{للطرفين}]{\text{اضافة } (-1)} \frac{2x-5}{x-2} - 1 \leq 0$ ترحيد المقامات

$\frac{2x-5-(x-2)}{x-2} \leq 0 \Rightarrow \frac{2x-5-x+2}{(x-2)} \leq 0 \Rightarrow \frac{x-3}{x-2} \leq 0$

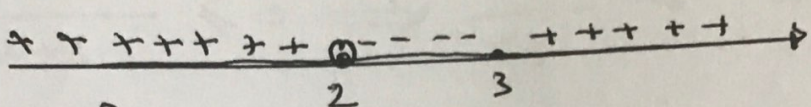
Sign of $(x-3)$



Sign of $(x-2)$



$\frac{(x-3)}{(x-2)}$



since $\frac{x-3}{x-2}$ must be negative

The solution set is

$\{x : 2 < x \leq 3\}$

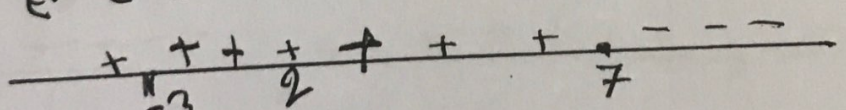
⑦ $\frac{1}{x-2} > \frac{2}{x+3} \Rightarrow \frac{1}{x-2} - \frac{2}{x+3} > 0$

$\Rightarrow \frac{(x+3)-2(x-2)}{(x-2)(x+3)} > 0$

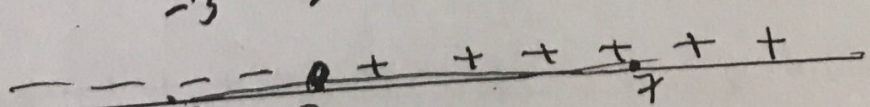
$\Rightarrow \frac{x+3-2x+4}{(x-2)(x+3)} > 0 \Rightarrow \frac{(7-x)}{(x-2)(x+3)} > 0$

$-(x-7) = (7-x)$

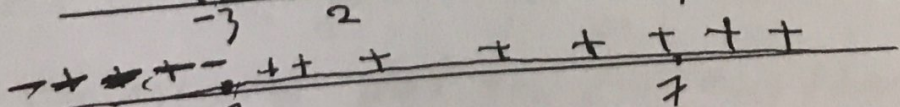
sign $(7-x)$



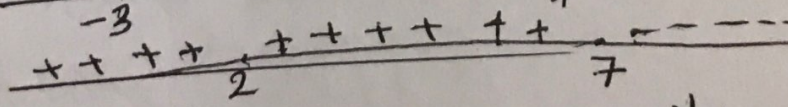
sign of $(x-2)$



sign of $(x+3)$



$\frac{(7-x)}{(x-2)(x+3)}$



since $\frac{(7-x)}{(x-2)(x+3)}$ must be positive then