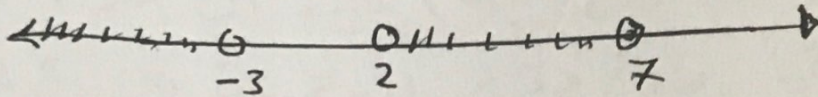


The solution set is

$$\{x: x < -3\} \cup \{x: 2 < x < 7\} = (-\infty, -3) \cup (2, 7)$$



⑧

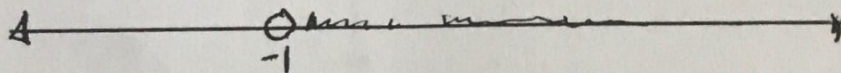
$$\frac{x-2}{x+1} \leq 1 \Rightarrow \frac{x-2}{x+1} - 1 \leq 0 \Rightarrow \frac{x-2-(x+1)}{x+1} \leq 0$$

$$\Rightarrow \frac{x-2-x-1}{x+1} \leq 0 \Rightarrow \frac{-3}{x+1} \leq 0 \xrightarrow{(-\frac{1}{2})x \cup \frac{3}{2}}$$

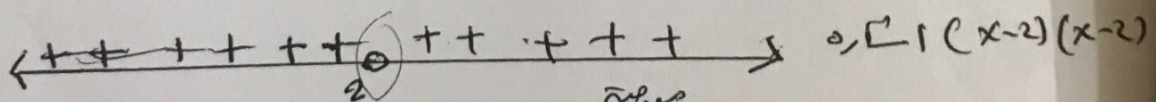
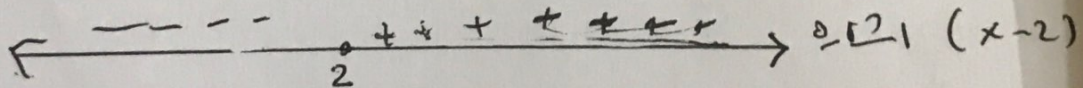
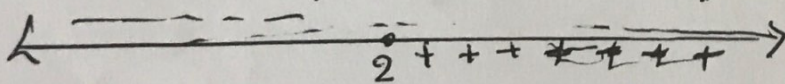
$$\frac{1}{x+1} \geq 0$$

$$\because 1 > 0 \Rightarrow x+1 > 0 \Rightarrow x > -1$$

$\therefore$  The solution set is  $\{x: x > -1\} = (-1, \infty)$



⑨  $(x-2)^2 > 0 \Rightarrow (x-2)(x-2) > 0$  Sign (x-2)



since  $(x-2)(x-2)$  must be positive, then

The solution is  $\mathbb{R} \setminus \{2\}$

(13)



# Absolute Value ( القيمة المطلقة )

Def of  $|x|$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Ex ①  $|6| = 6$

②  $|-6| = 6$

③  $|0| = 0$

properties of absolute value

الخواص

①  $|x| = 0$  if and only if  $x = 0$

②  $|x| = \sqrt{x^2}$  and  $|x|^2 = x^2$

③  $|-x| = |x|$

④  $|xy| = |x||y|$

⑤  $|\frac{x}{y}| = \frac{|x|}{|y|}$   $y \neq 0$

⑥  $-|x| \leq x \leq |x|$

⑦  $|x+y| \leq |x| + |y|$

⑧  $|x| \leq |y|$  if and only if  $x^2 \leq y^2$

Remark

①  $|a|$  = distance between  $a$  and  $0$   
②  $|a-c|$  = distance between  $a$  and  $c$



Remark: For any real number  $x$  and  $c$  and any positive real number  $a$

$$\textcircled{1} |x| < a \iff -a < x < a$$

$$-a < x - c < a$$

$$\textcircled{2} |x - c| < a \iff c - a < x < c + a$$

$$\textcircled{3} |x| > a \iff x > a \text{ or } x < -a$$

Example

Solve  $|2x - 3| = 7$

الحل Sol.

بـ تعريف القيمة المطلقة

$$|2x - 3| = \begin{cases} 2x - 3 & x \geq 0 \\ -(2x - 3) & x < 0 \end{cases}$$

if  $x \geq 0$

$$2x - 3 = 7$$

↓

$$2x = 10$$

↓

$$x = 5$$

if  $x < 0$

$$-(2x - 3) = 7$$

$$-2x + 3 = 7$$

↓

$$-2x = 4$$

↓

$$x = -2$$

(16)



Ex.2  $|x-4| < 5$

Solution

صَبِّ التَّعْرِيفِ

$$-5 < x-4 < 5 \Rightarrow -1 < x < 9$$

∴ The solution set is  $\{x: -1 < x < 9\} = (-1, 9)$

Ex.3  $|2x+7| \geq 1 \Rightarrow$

Sol.

صَبِّ التَّعْرِيفِ

$$2x+7 \geq 1$$

or

$$2x+7 \leq -1$$

↓

↓

$$2x \geq -6$$

or

$$2x \leq -8$$

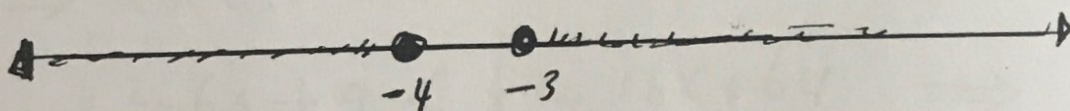
↓

↓

$$x \geq -3$$

$$x \leq -4$$

∴ The sol. set is  $\{x: x \geq -3\} \cup \{x: x \leq -4\} = [-3, \infty) \cup (-\infty, -4]$



Ex.4 Solve  $\frac{3}{|2x-1|} \geq 4$

Sol

$$\frac{3}{|2x-1|} \geq 4$$

صَبِّ التَّعْرِيفِ

↓

$$\frac{|2x-1|}{3} \leq \frac{1}{4} \Rightarrow |2x-1| \leq \frac{3}{4}$$

صَبِّ التَّعْرِيفِ

$$-\frac{3}{4} \leq 2x-1 \leq \frac{3}{4} \Rightarrow -\frac{3}{4}+1 \leq 2x \leq \frac{3}{4}+1$$

(17)

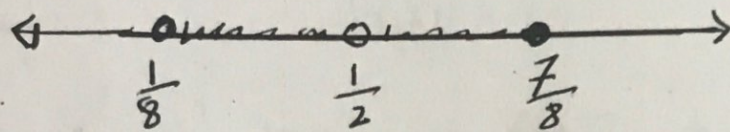


$$\Rightarrow \frac{1}{4} \leq 2x \leq \frac{7}{4} \Rightarrow \frac{1}{8} \leq x \leq \frac{7}{8}$$

∴  $x \neq \frac{1}{2}$  لأن  $x = \frac{1}{2}$  غير معرف عند  $x = \frac{1}{2}$  كانت هذه النقطة من مجموعة الحل

∴ The solution set is

$$\left[ \frac{1}{8}, \frac{1}{2} \right) \cup \left( \frac{1}{2}, \frac{7}{8} \right]$$



Ex  $|x+3| < |x-8|$

Sol  $|x+3| < |x-8| \Rightarrow \underline{|x+3|^2 < |x-8|^2}$

$$\Rightarrow (x+3)^2 < (x-8)^2 \Rightarrow$$

$$\underline{x^2 + 6x + 9} < \underline{x^2 - 16x + 64} \Rightarrow$$

$$\underline{22x} < \underline{55} \Rightarrow x < \frac{55}{22} \Rightarrow x < \frac{5}{2}$$

∴ The sol. set is

$$\{ x : x < \frac{5}{2} \} = (-\infty, \frac{5}{2})$$