3.2. Solution of Simultaneous equations with Matrices

Consider the relation

$$AX = B \quad (3.31)$$

where A and B are matrices whose elements are known, and X is a matrix (a column vector) whose elements are the unknowns. We assume that A and X are conformable for multiplication. Multiplication of both sides of (3.31) by yields:

$$A^{-1}AX = A^{-1}B = IX = A^{-1}B \quad (3.31)$$

or

$$X = A^{-1}B \quad (3.32)$$

Therefore, we can use (3.32) to solve any set of simultaneous equations that have solutions. We will refer to this method as the *inverse matrix method of solution* of simultaneous equations.

Example 3.11.1

Given the system of equations $\begin{Bmatrix} 2x_1 & +3x_2 & +x_3 & = 9 \\ x_1 & +2x_2 & +3x_3 & = 6 \\ 3x_1 & +x_2 & +2x_2 & = 8 \end{Bmatrix}$, compute the unknowns x_1, x_2, and $x_3$using the inverse matrix method

Solution

In matrix form, the given set of equations is $AX = B$ where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}, \qquad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad B = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

Then

$$X = A^{-1}B$$

Or

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

Next, we find the determinant $detA$, and the $adjA$.

$$detA = 18 \quad \text{and} \quad adjA = \begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{bmatrix}$$

Thus,

$$A^{-1} = \frac{1}{detA}\, adjA = \frac{1}{18}\begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{bmatrix}$$

and by equation (4.53) we obtain the solution as follows.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 1 & -5 & 7 \\ 7 & 1 & -5 \\ -5 & 7 & 1 \end{bmatrix}\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 35 \\ 29 \\ 5 \end{bmatrix} = \begin{bmatrix} 35/18 \\ 29/18 \\ 5/18 \end{bmatrix} = \begin{bmatrix} 1.94 \\ 1.61 \\ 0.28 \end{bmatrix}$$

To verify our results, we could use the MATLAB inv(A) function, and multiply $A^{-1}$by B. However, it is easier to use the *matrix left division* operation $X = A \setminus B$; this is MATLAB's solution of$A^{-1}B$ for the matrix equation $AX = B$, where matrix X is the same size as matrix B, see [1].

Check with MATLAB

Try it! In command window

```
>> A= [2 3 1; 1 2 3; 3 1 2]; B = [9; 6; 8]; X= A\ B  % Observe that B is column vector
 X =
     1.9444
     1.6111
     0.2778
```

Example 3.11.2

Consider that electric circuit is shown in the following figure

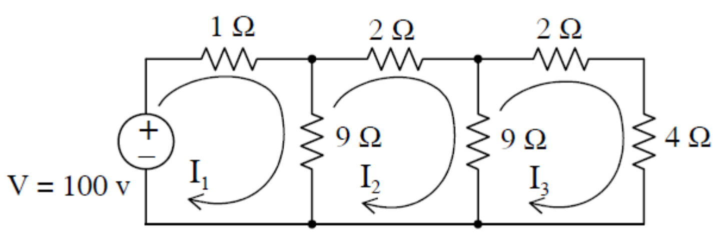

Figure 3.11.1: Electric circuit for example 3.11.2

The mesh equations are given as

$$\begin{aligned} 10I_1 - 9I_2 \qquad &= 100 \\ -9I_1 + 20I_2 - 9I_3 &= 0 \\ -9I_2 + 15I_3 &= 0 \end{aligned}$$

Use the inverse matrix method to compute the values of the currents I_1, I_2 and I_3.

Solution

For this example, the matrix equation is $RI = V$ or $I = R^{-1}V$, where

$$R = \begin{bmatrix} 10 & -9 & 0 \\ -9 & 20 & -9 \\ 0 & -9 & 15 \end{bmatrix}, \quad V = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}, \text{ and } \quad I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

The next step is to find R^{-1}. This is found from the relation

$$R^{-1} = \frac{1}{detR} adjR$$

Therefore, we find the determinant and the adjoint of R. For this example, we find that

$$detR = 975, \quad adjR = \begin{bmatrix} 219 & 135 & 81 \\ 135 & 150 & 90 \\ 81 & 90 & 119 \end{bmatrix}, \text{ then}$$

$$R^{-1} = \frac{1}{detR} adjR = \frac{1}{975} \begin{bmatrix} 219 & 135 & 81 \\ 135 & 150 & 90 \\ 81 & 90 & 119 \end{bmatrix}, \text{ and hence,}$$

$$I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{975} \begin{bmatrix} 219 & 135 & 81 \\ 135 & 150 & 90 \\ 81 & 90 & 119 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} = \frac{100}{975} \begin{bmatrix} 219 \\ 135 \\ 81 \end{bmatrix} = \begin{bmatrix} 22.46 \\ 13.85 \\ 8.31 \end{bmatrix}$$

Type equation here.

Check with MATLAB

Try it! In command window

```
>> R= [10 -9 0; -9 20 -9; 0 -9 15]; V = [100; 0; 0]; I= R\ V  % Observe that B is column vector
 I =
    22.4615
    13.8562
    8.31077
```