Chapter Two
(Binding Energy)

(2-1) *Nuclear Binding Energy*

Since an atom contains Z positively charged particles (protons) and N=A-Z neutral particles (neutrons), the total charge of a nucleus is +Ze, where e represents the charge of one electron. Thus, the mass of a neutral atom, M_{atom}, can be expressed in terms of the mass of its nucleus, M_{nuc} and its electrons m_e.

$$M_{atom} = M_{nuc} + Zm_e \qquad M_{nuc} = Zm_p + (A-Z)m_n$$

where m_p is the proton mass, m_e the mass of an electron and m_n the mass of a neutron. For example the mass of the rubidium nucleus, ^{87}Rb, which contains 37 protons and 50 neutrons, can be calculated as:

$$M_{nuc}(^{87}Rb) = 37\times1.007277 + 50\times1.008665 = 87.7025amu$$

The atomic mass, indicated on most tables of the elements, is the sum of the nuclear mass and the total mass of the electrons present in a neutral atom. In the case of ^{87}Rb, 37 electrons are present to balance the charge of the 37 protons. The atomic mass of ^{87}Rb is then:

$$M_{atom}(^{87}\text{Rb}) = M_{nuc}(^{87}\text{Rb}) + Zm_e$$
$$= 87.7025 + 37\times0.00055 = 87.7228amu$$

From the periodic table, the measured mass of a ^{87}Rb atom is found to be $M_A^{measured}(^{87}Rb)$ = 86.909187 amu. These two masses are not equal and the difference is given by:

$$\Delta m = M_{atom}(^{87}Rb) - M_{atom}^{measured}(^{87}Rb) = 0.813613 amu$$

Expanding the terms in this equation, shows that the difference in mass corresponds to a difference in the mass of the nucleus

$$\Delta m = M_{atom} - M_{atom}^{measured}$$
$$= Zm_p + Zm_e + (A-Z)m_n - M_{nuc}^{measured} - Zm_e$$

which reduces to

$$\Delta m = M_{atom} - M_{atom}^{measured}$$
$$= Zm_p + (A-Z)m_n - M_{nuc}^{measured} = M_{nuc} - M_{nuc}^{measured}$$

Thus, when using atomic mass values given by the periodic table, the mass difference between the measured and calculated is given by

$$\Delta m = M_{nuc} - M_{nuc}^{measured} = Zm_p + Zm_e + (A-Z)m_n - M_{atom}^{measured}$$

Notice also that

$$Zm_p + Zm_e = Zm_H$$

Where m_H is a mass of the hydrogen atom.

From this and other examples it can be concluded that the actual mass of an atomic nucleus is always smaller than the sum of the rest masses of all its nucleons (protons and neutrons). This is because some of the mass of the nucleons is converted into the energy that is needed to form that nucleus and hold it together. This converted mass, Δm, is called the “mass defect” and the corresponding energy is called the “binding energy” and is related to the stability of the nucleus; the greater binding energy leads to the more stable the nucleus. This energy also represents the minimum energy required to separate a nucleus into protons and neutrons. The mass defect and binding energy can be directly related, as shown below:

$$B(A,Z) = \Delta m \times 931.5 MeV/amu \qquad \text{or}$$

$$B(A,Z) = 931.5(Zm_p + Zm_e + Nm_n - M_{atom}^{measured})$$

Since the total binding energy of the nucleus depends on the number of nucleons, a more useful measure of the cohesiveness is the average binding energy B_{ave}.

$$B_{ave}(A,Z) = \frac{B(A,Z)}{A} \quad (MeV/nucleon)$$

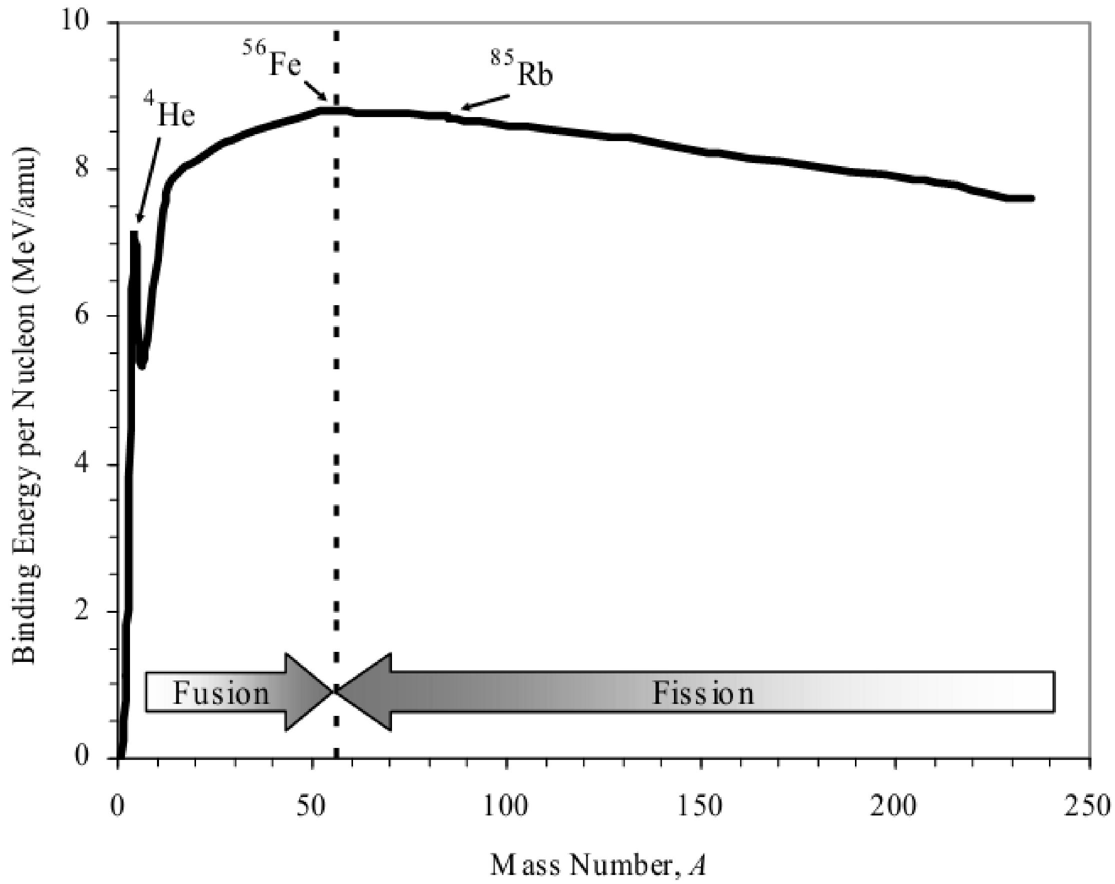

Figure (2-1): Variation of binding energy per nucleon with the atomic mass number

The binding energy per nucleon varies with the atomic mass number A, as shown in figure (2-1). For example, the binding energy per nucleon in a rubidium nucleus is 8.7MeV, while in helium it is 7.3MeV. The curve indicates three characteristic regions:

- Region of stability: A flat region between (A) equal to approximately 35 and 70 where the binding energy per nucleon is nearly constant. This region exhibits a peak near A = 60. These nuclei are near iron and are called the iron peak nuclei representing the most stable elements.
- Region of fission reactions: From the curve it can be seen that the heaviest nuclei are less stable than the nuclei near A = 60, which suggests that energy can be released if heavy nuclei split apart into smaller nuclei having masses nearer the iron peak. This process is called fission (the basic nuclear reaction used in atomic bombs as uncontrolled reactions and in nuclear power and research reactors as controlled chain reactions). Each fission event generates nuclei commonly referred to as fission fragments with mass numbers ranging from 80 to 160.
- Region of fusion reactions: The curve of binding energy suggests a second way in which energy could be released in nuclear reactions. The lightest elements (like hydrogen and helium) have nuclei that are less stable than heavier elements up to the iron peak. If two light nuclei can form a heavier nucleus a significant energy could be released. This process is called fusion, and represents the basic nuclear reaction in hydrogen (thermonuclear) weapons.

(2-2) ***Separation energy***

Are the analogous of the ionization energies in atomic physics, reflecting the energies of the valence nucleons. The separation energy of any particle is defined as the amount of energy needed to remove a particle from the nucleus. For a given N,Z; S_n , S_p is larger for nuclei with even N or Z than with odd one, this due to the pair effect of nuclear force which increase the binding energy and separation energy.

There are two equations to determine the separation energy using the mass or the binding energy. For neutron as follow:

$S_n=931.5[M(A-1,Z)+m_n-M(A,Z)]$

$S_n=B(A,Z)-B(A-1,Z)$

For proton:

$S_p=931.5[M(A-1,Z-1)+m_H-M(A,Z)]$

$S_p=B(A,Z)-B(A-1,Z-1)$

For alpha particle:

$S_\alpha=931.5[M(A-4,Z-2)+m_\alpha-M(A,Z)]$

$S_\alpha=B(A,Z)-B(A-4,Z-2)-B(\alpha\equiv{}^4_2He)$

The fact, that each pair of these equations represents two equivalent equations.

Example: calculate the separation energy of neutron for ^{209}Pb by using the two methods, where $M({}^{209}_{82}Pb)$=209.05398u, $M({}^{208}_{82}Pb)$=208.04754u.

Sol.:

1- $S_n=931.5[M(A-1,Z)+m_n-M(A,Z)]= 931.5[M({}^{208}_{82}Pb)+m_n-M({}^{209}_{82}Pb)]$

$=931.5[208.04754+1.008665-209.05398]=2.07259MeV$

2- $B(A,Z)=931.5(Zm_p+Zm_e+Nm_n-M^{measured}_{atom})$

$B({}^{209}_{82}Pb)=931.5(82x1.007276+82x0.000549+127x1.008665-209.05398]$

=1572.48844MeV

B($^{208}_{82}Pb$)=931.5(82x1.007276+82x0.000549+126x1.008665-208.04754]

=1570.41585MeV

S_n=B(A,Z)-B(A-1,Z)=1572.48844-1570.41585=2.07259MeV

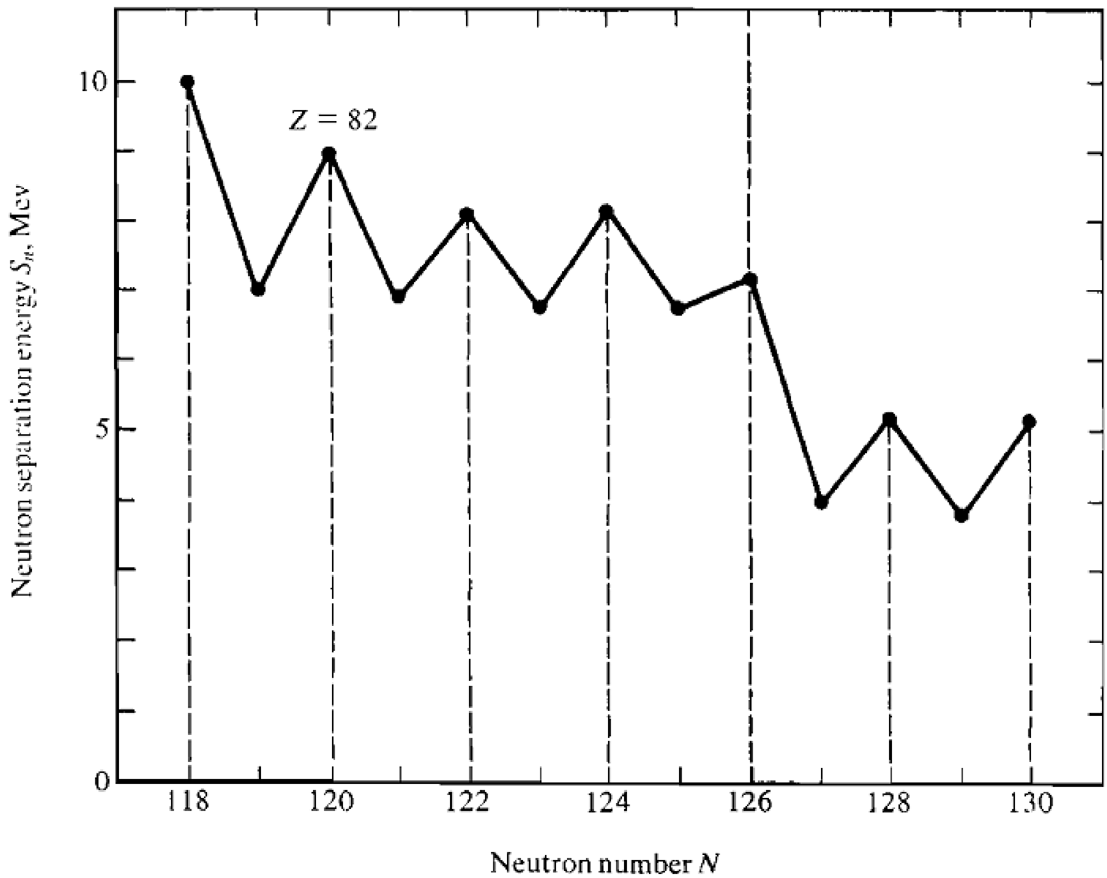

Figure (2-2): Neutron separation energy of lead isotopes as a function of neutron number.

(2-3) *Nuclear Forces*

Protons and neutrons are bound inside nuclei, despite the Coulomb repulsion among protons. Therefore there must be different and much stronger force acting among nucleons to bind them together. This force is called nuclear force, nuclear binding force, or in more modern settings, the strong interaction. There are notable properties of the nuclear binding force.

1. It is much stronger than the electromagnetic force (the force is charge independent, i.e. $F_{pp}=F_{nn}=F_{pn}$, we can see that from the equality of energy level, binding energy and total angular momentum of mirror nuclei). As shown in the empirical mass formula [see section (2-5-1)], the coefficient of the Coulomb term is more than an order of magnitude smaller than the other terms in the binding energy.

2. It is short-ranged, acts only up to 1–2 fm.

3. It has the saturation property, giving nearly constant B/A= $B_{ave} \approx 8.5$ MeV.

This is in stark contrast to the electromagnetic force.

4. The force depends on spin and states of the nucleon.

i.e. the nuclear force between two nucleons of the same type (p and p) or (n and n) could be the biggest whenever the total angular momentum for the first has the maximum value and equal with opposite direction to the other, i.e. the angular momentum for both is equal to zero.

For example, let $\vec{s}_1$ and $\vec{s}_2$ are the spin to the two protons, $\vec{\ell}_1$ and $\vec{\ell}_2$ the orbital momentum, therefore the total angular momentum for the first is equal $\vec{j}_1 = \vec{\ell}_1 + \vec{s}_1$, and for the second proton $\vec{j}_2 = \vec{\ell}_2 + \vec{s}_2$

For the maximum nuclear force between the two nucleons, must be $\vec{j}_{1max} = -\vec{j}_{2max}$

Put the two protons in s-state, then:

$\vec{\ell}_1 = 0, \quad \vec{s}_1 = 1/2 \Rightarrow \vec{j}_1 = 1/2$ and $\vec{\ell}_2 = 0, \quad \vec{s}_2 = 1/2 \Rightarrow \vec{j}_2 = 1/2$

For maximum F_{pp}, $\vec{j}_1 = -\vec{j}_2 \Rightarrow \vec{J} = \vec{j}_1 + \vec{j}_2 = 1/2 - 1/2 = 0$

for two protons in p-state, then:

$\vec{\ell}_1 = 1, \quad \vec{s}_1 = 1/2 \Rightarrow \vec{j}_{1\min} = 1/2$ (if $\vec{\ell}_1 \# \vec{s}_1$) and $\vec{j}_{1\max} = 3/2$ (if $\vec{\ell}_1 // \vec{s}_1$)

$\vec{\ell}_2 = 1, \quad \vec{s}_2 = 1/2 \Rightarrow \vec{j}_{2\min} = 1/2$ (if $\vec{\ell}_2 \# \vec{s}_2$) and $\vec{j}_{2\max} = 3/2$ (if $\vec{\ell}_2 // \vec{s}_2$)

For maximum F_{pp}, $\vec{j}_{1\max} = -\vec{j}_{2\max} \Rightarrow \vec{J} = \vec{j}_{1\max} + \vec{j}_{2\max} = 3/2 - 3/2 = 0$

This phenomenon is called the pairing effect.

5. It is exchange forces. Like of the photon exchange between the electric charges, there are medium mass particles (mesons) were exchange between nucleons.

6. Even though the nuclear force is attractive to bind nucleons, there is a repulsive core when they approach too closely, around 0.5fm. They basically cannot go closer.

i.e. 2fm>r>0.5fm leads to attractive nuclear force, while r<0.5fm repulsive force.

(2-4) *Nuclear Spins and Dipole Moments*

Both the proton and the neutron have spin angular momentum of $\frac{1}{2}\hbar$.

Furthermore, just as electrons in an atom can have orbital angular momentum, so also can nucleons inside a nucleus. We know from quantum mechanics that orbital angular momentum can take on only integral values. The total angular momentum of the constituents-namely, the vector sum of the orbital and intrinsic spin angular momenta-defines the spin of the nucleus.

Thus, the nuclei with even mass number have integral nuclear spin whereas nuclei with odd mass number have half-integral nuclear spin. However, the nuclei with an even number of protons and an even number of neutrons (even-even nuclei) have zero nuclear spin. These facts lend credence to the hypothesis that spins of nucleons inside a nucleus are very strongly paired so as to cancel their overall effect.

To explain the fine structure of the spectral lines, suppose that each of the electron, proton and neutron has spin momentum result from rotation on its axis and therefore they has a magnetic moment due to this rotation, the interaction of magnetic moment of the electron with the magnetic moment of the nucleus leads to increase or decrease the tension between them and then will increase or decrease energy of electron, i.e. split the energy levels of the electron and thus will be divided every line of spectral lines into several lines. Every charged particle has a magnetic dipole moment associated with its spin, given by:

$$\vec{\mu}_s = g_s \frac{e}{2mc} \vec{s}$$

where e, m and s are the charge, mass and the intrinsic spin of the charged particle. The constant g is known as the Lande factor (Gyromagnetic ratio), which for a point particle, such as the electron, is expected to have the value $g = 2$.

When $g \neq 2$ the particle is said to possess an anomalous magnetic moment, which is usually ascribed to the particle having a substructure like proton and neutron:

$$g_s \approx \begin{cases} 5.5857 & \text{proton} \\ -3.8261 & \text{neutron} \end{cases}$$

Relate to orbital angular momentum

$$\vec{\mu}_\ell = g_\ell \frac{e}{2mc} \vec{\ell}$$

$$g_\ell = 1 \quad \text{for proton}$$
$$g_\ell = 0 \quad \text{for neutron}$$

For the electron (with $s = \frac{1}{2}\hbar$) , the dipole moment $\mu_e \approx \mu_B$, where μ_B is the Bohr magneton, defined as:

$$\mu_B = \frac{e\hbar}{2m_e c} = 5.79 \times 10^{-11}\,\text{MeV/T}$$

Where a magnetic field of 1 tesla (T) corresponds to 10^4 gauss (G), the magnetic dipole moment for nucleons is measured in terms of the nuclear magneton, defined using the proton mass:

$$\mu_N = \frac{e\hbar}{2m_p c} = 3.15 \times 10^{-14}\,\text{MeV/T}$$

From the ratio of m_p/m_e , we deduce that the Bohr magneton is about 2000 times larger than the nuclear magneton due to $m_p = 1837 m_e \approx 2000 m_e$, i.e. atomic moment >> nuclear moment.

The magnetic moments of the proton and the neutron are:

$$\mu_p \approx 2.79\mu_N \quad , \quad \mu_n \approx -1.91\mu_N$$

Thus, the electrons cannot be present inside nuclei because it would then be particularly hard to explain the small values of nuclear moments, since even one electron would produce a moment a thousand times that observed for nuclei.

The most important Radionuclides:

1. **Hydrogen (Deuterium & Tritium)** $^{2}_{1}H$ **&** $^{3}_{1}H$
2. **Helium (Alpha particle)** $^{4}_{2}He$
3. **Carbon** $^{14}_{6}C$
4. **Sodium** $^{22}_{11}Na$
5. **Potassium** $^{40}_{19}K$
6. **Cobalt** $^{60}_{27}Co$
7. **Strontium** $^{90}_{38}Sr$

8. **Iodine $^{131}_{53}I$**
9. **Cesium $^{137}_{55}Cs_{55}$Cs**
10. **Barium $^{133}_{56}Ba$**
11. **Europium $^{152}_{63}Eu$**
12. **Lead $^{214}_{82}Pb$**
13. **Bismuth $^{214}_{83}Bi$**
14. **Polonium $^{210}_{84}Po$**
15. **Radon $^{222}_{86}Rn$**
16. **Radium $^{226}_{88}Ra$**
17. **Thorium $^{232}_{90}Th$**
18. **Uranium $^{238}_{92}U$ & $^{235}_{92}U$**
19. **Neptunium $^{237}_{93}Np$**
20. **Plutonium $^{239}_{94}Pu$**
21. **Americium $^{241}_{95}Am$**