Al-Mustansiriyah University
College of Science
Physics Department

Fourth Grade
Nuclear Structure
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Semester-1 (Nuclear structure syllabus)

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2- Stability and Abundance
3- Nuclear Mass, Charge, Size and Density
4- Quantum theory of angular momentum

Chapter Two (Binding Energy)

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Chapter One

(Nuclear Concepts)

(1-1) History

The main stages of the nuclear physics are the following:

- 1868 Mendeleev's periodic classification of the elements.
- 1895 Discovery of X-rays by Roentgen.
- 1896 Discovery of radioactivity by Becquerel.
- 1897 Identification of the electron by J.J. Thomson.
- 1898 Separation of the elements polonium and radium by Pierre and Marie Curie.
- 1908 Measurement of the charge +2 of α particle by Geiger and Rutherford.
- 1911 Discovery of the nucleus by Rutherford; "planetary" model of the atom.
- 1913 Theory of atomic spectra by Niels Bohr.
- 1914 Measurement of the mass of α particle by Robinson and Rutherford.
- 1924–1928 Quantum theory (de Broglie, Schrodinger, Heisenberg, Born, Dirac).
- 1928 Theory of barrier penetration by quantum tunneling, application to α radioactivity, by Gamow, Gurney and Condon.

- 1929–1932 First nuclear reactions with the electrostatic accelerator of Cockcroft and Walton and the cyclotron of Lawrence.
- 1930–1933 Neutrino proposed by Pauli and named by Fermi in his theory of beta decay.
- 1932 Identification of the neutron by Chadwick.
- 1934 Discovery of artificial radioactivity by Joliot-Curie.
- 1934 Discovery of neutron capture by Fermi.
- 1935 Liquid-drop model and compound nucleus model of N. Bohr.
- 1935 Semi-empirical mass formula of Bethe and Weizsacker.
- 1938 Discovery of fission by Hahn and Strassman.
- 1939 Theoretical interpretation of fission by Meitner, Bohr and Wheeler.

To these fundamental discoveries we should add the practical applications of nuclear physics. Apart from nuclear energy production beginning with Fermi's construction of the first fission reactor in 1942, the most important are astrophysical and cosmological. Among them are:

- 1938 Bethe and Weizsacker propose that stellar energy comes from thermonuclear fusion reactions.
- 1946 Gamow develops the theory of cosmological nucleosynthesis.
- 1953 Salpeter discovers the fundamental solar fusion reaction of two protons into deuteron.
- 1957 Theory of stellar nucleosynthesis by Burbidge, Fowler and Hoyle.
- 1960 Detection of solar neutrinos
- 1987 Detection of neutrinos and γ-rays from the supernova.

(1-2) Introduction

Nuclei sit at the center of any atoms. Therefore, understanding them is of central importance to any discussions of microscopic physics. As you know, nuclei are composed of protons and neutrons. The number of protons is the atomic number (Z), and the mass number (A) is equal to the total number of nucleons (a collective name for protons and neutrons), Therefore, A = N + Z where (N) is the number of neutrons. Isotopes are denoted by ${}^{A}_{Z}\mathbf{X}_{N}$ or more often by ${}^{A}_{Z}\mathbf{X}$ or just ${}^{A}\mathbf{X}$, where X is the chemical symbol and A is the mass number, for example ${}^{238}_{92}\mathbf{U}_{146}$.

(1-3) Definitions and Units

Nuclide = nucleus with a specific N and Z (e.g. ${}^{14}_{6}\mathbf{C}_{8}$)

Isotope = two nuclei with same Z and different N. (e.g. ${}^{12}_{6}\mathbf{C}_{6}$, ${}^{14}_{6}\mathbf{C}_{8}$)

Isotone = two nuclei with same N and different Z (e.g. ${}^{12}_{6}\mathbf{C}_{6}$, ${}^{14}_{8}\mathbf{O}_{6}$)

Isobar = two nuclei with same A, different Z and N (e.g. ${}^{14}_{7}\mathbf{N}_{7}$, ${}^{14}_{6}\mathbf{C}_{8}$)

Isomer = same isotope but with excited state (usually long-lived)
(e.g. ^{189}Au, stable; ^{189m}Au, half-life = 4 minutes)

Mirror nuclei are isobars (same A) with opposite numbers of protons and neutrons.

For example, ^{14}C (Z=6, N=8) and ^{14}O (Z=8, N=6) are mirror nuclei.

Abundance = relative percentage (by number) of isotope.

Photon is quantity of electromagnetic energy with a specific linear momentum.

The equations which applied to each particle moving with light speed, as follow:

$E=mc^2$ (photon as a particle "Einstein equation")

$E=h\nu=hc/\lambda$ (photon as a wave "plank equation")

$P=mc$ (photon as a particle)

$P=h/\lambda=h\nu/c$ (photon as a wave "Compton assumption")

Units:

Length: 1 angstrom = 10^{-10}m = 1Å

1 fermi (or femtometer) = 10^{-15}m = 1fm.

Energy: 1 electron volt (eV) = energy of electron accelerated through 1 volt electrical potential = 1.6×10^{-19}J.

1u (atomic mass unit) = 931.502MeV/c^2 (where ^{12}C has mass = 12.00000u)

m_p = 938.280MeV/c^2 m_n= 939.573MeV/c^2 m_e = 0.511MeV/c^2

Speed of light, c = $3\text{x}10^8$m/s, Electron charge, e = $1.6\text{x}10^{-19}$C,

Planck constant, h = $6.63\text{x}10^{-34}$J.s, Avogadro's number, N_a = $6.022\text{x}10^{23}\text{mol}^{-1}$

(1-4) Stability and Abundance

When we examine the characteristics of stable nuclei, we find that for A<40 the number of protons equals the number of neutrons (N=Z). But for A≥40, stable nuclei have N=1.7Z; i.e., the number of neutrons is greater than the number of protons, see figure (1-1). This can be understood from the fact that, in larger nuclei, the charge density, and therefore the destabilizing effect of Coulomb repulsion, is smaller when there is a neutron excess. Furthermore, a survey of the stable nuclei reveals that even-even nuclei are the ones most abundant in nature. This again lends support to the strong-pairing hypothesis, namely that pairing of nucleons leads to nuclear stability. The most stability of nuclei has a magic numbers (2, 8, 20, 28, 50, 82, 126 and 184) of protons or neutrons to making a closed shell.

The numbers of stable nuclei in nature are:

For even-even = 156, even-odd = 48, odd-even = 50 and odd-odd = 5 (^{1}H, ^{6}Li, ^{10}B and ^{14}N).

The most important stability parameters:

1. Magic numbers
2. Pairing of nucleons
3. Equality and percentage between protons & neutrons
4. Mass number (A)

From these parameter, we can predict the stability and which of radiation may be emitting from the nucleus.

Example: How many atoms of ^{10}B are there in 5 grams of boron?

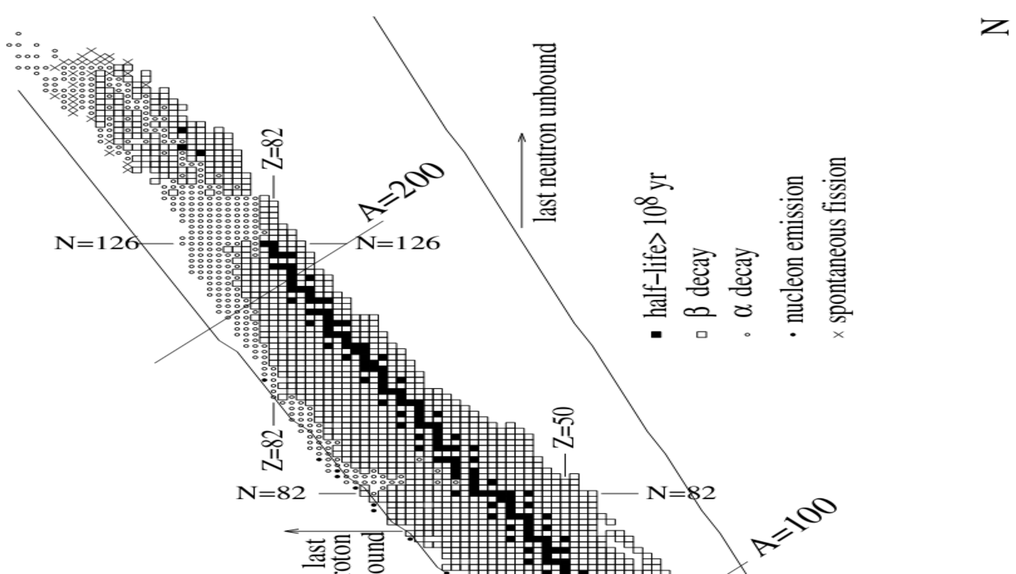
Sol.: From tables; the atomic weight of elemental boron W(B) = 10.811gm/mol. The 5gm sample of boron equals m/W(B) moles of boron, and since each mole contains N_a atoms, the number of boron atoms is:

$$N(B) = \frac{mN_a}{W(B)} = \frac{5gm(0.6022x10^{24} atoms/mol)}{10.811gm/mol} = 2.785x10^{23} atoms$$

From tables, the isotopic abundance of ^{10}B in elemental boron is found to be 19.9%. The number $N(^{10}B)$ of ^{10}B atoms in the sample is therefore,

$N(^{10}B) = (0.199)(2.785x10^{23}) = 5.542x10^{22}$ atoms

Or $\mathbf{N(B) \approx \frac{mN_a}{A}}$, where $\mathbf{W(B) \approx}$ **mass number (A)**



(1-5) Nuclear Mass and Charge

Nuclear masses are measured in terms of the atomic mass unit : 1amu or 1u= $1.66x10^{-27}$kg. One u is equivalent to 1/12 of the mass of a neutral ground-state atom of ^{12}C. Since electrons are much lighter than protons and neutrons (protons and neutrons have approximately similar mass), one nucleon has mass of about 1amu. Because of the mass-energy equivalence, we will often express masses in terms of energy units. To convert between energy (in MeV) and mass (in u) the conversion factor is of course the speed of light square (since $E = mc^2$). In these units we have: 1u = $931.502 MeV/c^2$.

- Proton mass: $m_p = 1.007276u = 938.280 MeV/c^2$
- Neutron mass: $m_n = 1.008665u = 939.573 MeV/c^2$
- Electron mass: $m_e = 0.000549u = 0.511 MeV/c^2$

Mass difference between proton and neutron of order one part per thousand of u or, $(m_n - m_p)c^2 = 1.29$ MeV

For nuclear physics, the mass difference is much more important than the masses themselves. Also of great phenomenological importance is the fact that this mass difference is of the same order as the electron mass.

$m_e c^2 = 0.511$ MeV

We would expect the mass of the nucleus to be:

$M_N(A,Z) \approx Zm_p + Nm_n$

For atoms, $M_A(A,Z) \approx Zm_p + Zm_e + Nm_n$

However, the measured values of nuclear masses revel that the mass of a nucleus is smaller than the sum of the masses of its constituents. Namely,

$M_N(A,Z) = Zm_p + Nm_n - B$

Where B=B.E. is the nuclear binding energy=mass difference (ΔM)

This explains why an isolated nucleus cannot just fall apart into its constituents, because that would violate the principle of conservation of energy.

The mass difference comes from the energy gained in bringing the nucleons into their mutual potentials, is the mass defect (Δ) which written as:

$\Delta = [M(A,Z) - A]c^2$

mass excess = [M(A,Z)-A]

$$\text{packing fraction} = \frac{M(A,Z) - A}{A}$$

Which is negative, and can be thought of as being proportional to the nuclear binding energy (B.E.); the absolute value of Δ is related to the minimum energy required to break up the nucleus into its components.

Example: for ^{16}O (Z=8, N=8)

From tables, Atomic mass= 15.994915 u =15.994915 x 931.502 = 14899.295 MeV/c^2

$\Delta=[M(A,Z)-A]c^2$

$=(15.994915 - 16)c^2 = -0.005085\ c^2 x 931.502 MeV/c^2 = -4.737 MeV$

Atomic mass= $8m_p+8m_e+8m_n$=15026.912 MeV/c^2

$\Delta M = [Zm_p + Zm_e + Nm_n - M(A,Z)]c^2$

ΔM=15026.912-14899.295=127.617 MeV

127.617/16=7.976 MeV per nucleon

Atomic nuclei are quantum bound states of particles called nucleons of which there are two types, the positively charged proton and the uncharged neutron. As far as we know, leptons are elementary particles that cannot be considered as bound states of constituent particles. Nucleons, on the other hand, are believed to be bound states of three spin 1/2 fermions called quarks. Two species of quarks, the up-quark u (charge 2/3) and the down quark d (charge -1/3) are needed to construct the nucleons:

proton = uud , neutron = udd .

$q_p = -e = +1.6\times10^{-19}$C, nuclear charge $Q_N = Zq_p$

Besides protons and neutrons, there exist many other particles that are bound states of quarks and antiquarks. Such particles are called hadrons. For nuclear physics, the most important are the three pions: (π^+, π^0, π^-), that strong interactions between nucleons result from the exchange of pions and other hadrons just like the electromagnetic interactions which results from the exchange of photons.

(1-6) Nuclear Size and Density

The existence of the nucleus as the small central part of an atom was first proposed by Rutherford in 1911. Later, in 1920, the radii of a few heavy nuclei were measured by Chadwick and were found to be of the order of 10^{-14}m, much smaller than the order of 10^{-10}m for atomic radii.

$R = r_o A^{1/3} = 1.2A^{1/3}$ fm

$$r_o \approx \begin{cases} 1.4\text{fm} & \text{for nuclear particle scattering on nuclei} \\ 1.2\text{fm} & \text{for electron scattering on nuclei} \end{cases}$$

The density of nuclei is approximately constant, and those nucleons are tightly packed inside the nucleus.

$$\rho \propto \frac{A}{R^3} \quad , \quad R = r_o A^{1/3} \Rightarrow \rho \propto \frac{1}{(r_o)^3} = constant$$

The experiments have been performed and analyzed for a great many nuclei and at several incident electron energies. All the result can be approximately explained by a charge distribution given by:

$$\rho(r) = \frac{\rho_o}{1 + \exp[(r - R)/a]}$$

Where the physical significance of the various parameters are illustrated in figure (1-2).

$\rho_o \approx 1.65x10^{44}$ nucleons/m^3 =0.165 nucleons/fm^3

$R \approx 1.07A^{1/3}$ fm, $a \approx 0.55$fm, t is the surface region

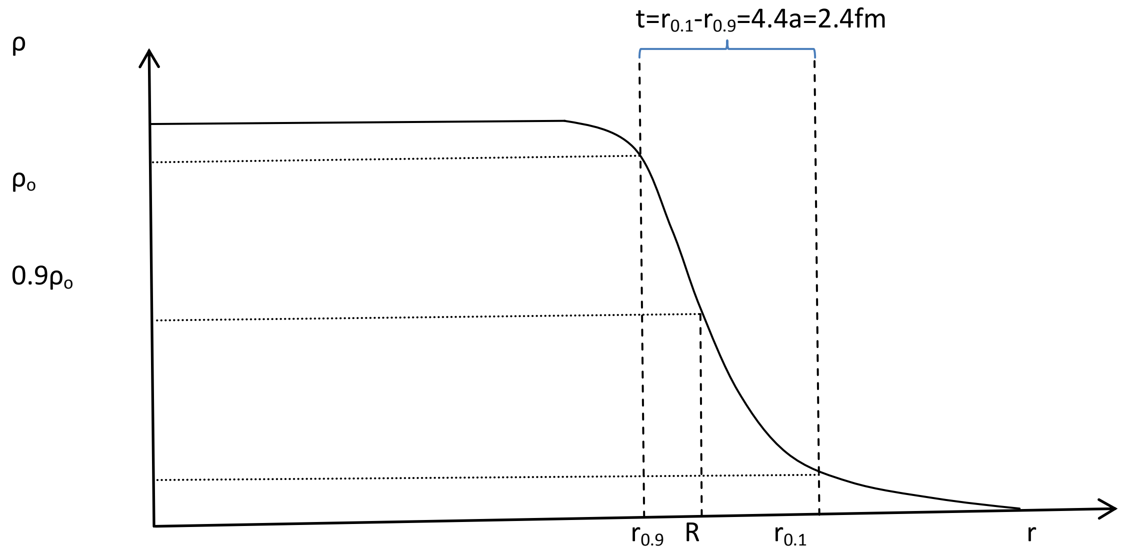


Figure (1-2): plot of nuclear density equation ρ(r) Vs. r, the meaning of ρ_o, R and a are illustrated.

From the distribution of the α-particle's scattering angles, Rutherford concluded that the structure of an atom most likely mimics the solar planetary system. The size of the nucleus at the center of the atom was estimated based on the kinetic energy (T) of the incident α-particle and its potential energy at the point of closest approach (d). The closest approach occurs in the case of a head-on collision in which the α-particle comes to rest before it bounces back at an angle of 180 degrees, see figure (1-3). At that point the kinetic energy is zero, and the potential energy equals the initial kinetic energy.

$$\mathbf{T = \frac{k(Ze)(ze)}{d}}$$

$$\mathbf{b = \frac{k(Ze)(ze)}{T\tan(\theta/2)} = \frac{d}{\tan(\theta/2)}}$$

Where k is the Coulomb force constant = 1 / ($4\pi\varepsilon_o$) = 8.99×10^9 Nm^2/C^2.

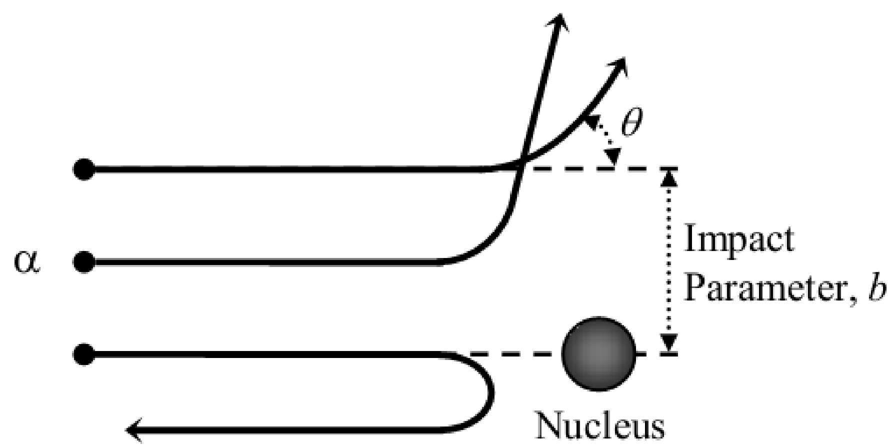


Figure (1-3): deflection of α particle by gold nucleus (of radius R).

Example: In Rutherford's experiment the kinetic energy of the incident α particles was 7.7MeV. Estimate the upper limit size of the gold nucleus and comment on the effect of increased energy of the incident particles in the experiment.

Sol.: the point of closest approach will determine the size of the nucleus. For the head-on collision it follows as:

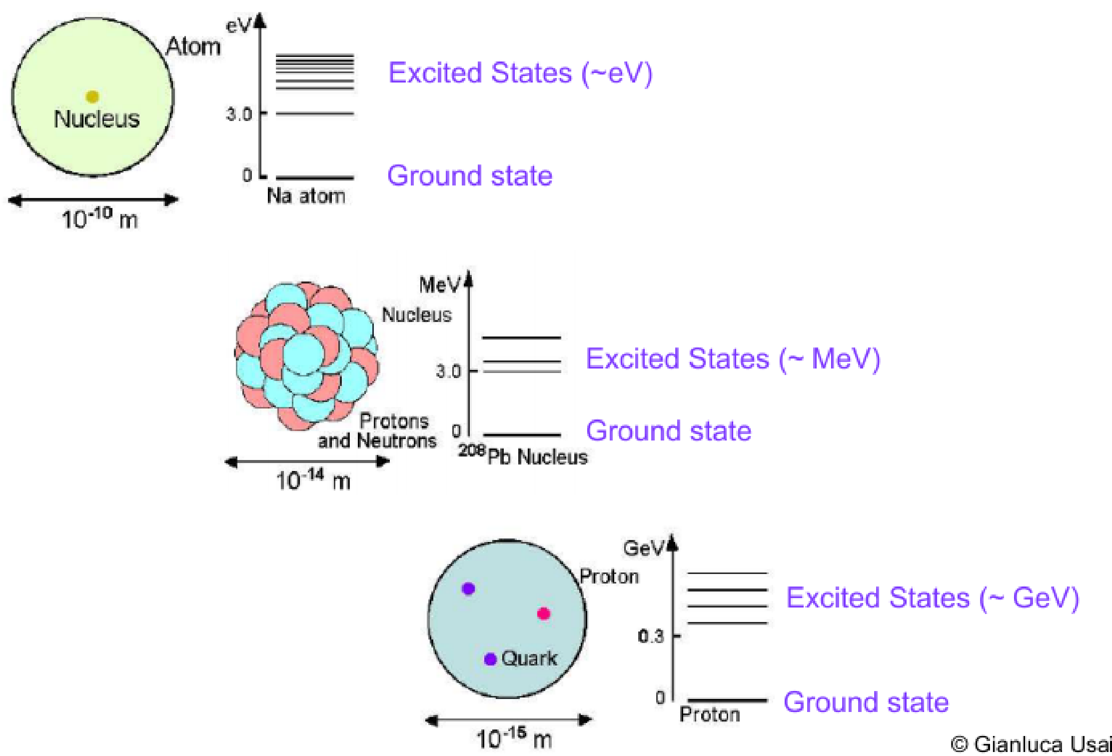
$$d = \frac{k(79e)(2e)}{7.7\,MeV} = \frac{(79)(2)ke^2}{7.7\,MeV} = 30\,fm$$

This implies that the gold nucleus has radius smaller than 30fm (the actual measurement is about 8fm).

If the incident energy of α particles in Rutherford's experiment is increased, some of the particles would penetrate the nucleus; first in the head-on collisions and then for smaller angles as the energy is further increased. The limiting kinetic energy for the incident particle above which the Rutherford experiment would not agree with theoretical explanation.

$$T \approx \frac{(79)(2)ke^2}{R} = 28.5\text{MeV}$$

Where R represents the radius of the gold nucleus.



(1-7) Quantum theory of angular momentum

Orbital angular momentum (from relative motion) is quantized in units of ħ, where ħ=h/2π, h is Planck's constant, L = 0 ħ, 1 ħ, 2 ħ, ...

Spin (intrinsic angular momentum) denoted by s for single particle and I for nucleus (nuclear spin), can be either integral or half-integral:

Fermions have half-integral spin s= 1/2 ħ, 3/2 ħ, 5/2 ħ, ...

Typical fermions include electrons, protons, neutrons, quarks, neutrinos...

Bosons have integral spin s = 0 ħ, 1 ħ, 2 ħ, ...

Typical bosons include pions, photons, W- and Z-bosons, gluons, (gravitons).

Since angular momentum is a vector, the total angular momentum of a nucleus is the vector sum of the angular momentum of its constituents, we find experimentally that complex nuclei have intrinsic angular momentum equal to I ħ, where:

For even-A nuclei: I is an integer (including zero)

For odd-A nuclei: I is an integer (including zero) plus one-half

For even-even nuclei: I=0

For example, the nucleus of deuterium ^{2}H has I=1 and the nucleus of ^{7}Li has I=3/2.

In QM we can only discuss the total angular momentum J and one component, usually J_z (The other components are in determinate), J_z can take on the values

J_z = -J, -J+1, -J+2, ... J-1, J i.e. (-J→J), sometimes for J_z we write m. So the angular momentum for a particle (or system of particles) is denoted by (J,J_z) or (J,m_ℓ).

$$\vec{J} = \sum_i (\vec{\ell}_i + \vec{s}_i) = \sum_i \vec{j}_i$$

$$\vec{J} = \vec{L} + \vec{S}$$

where $\vec{L} = \sum_i \vec{\ell}_i$ and $\vec{S} = \sum_i \vec{s}_i$

Adding angular momentum: The Rules; Suppose we start with (J_1,m_1) and (J_2,m_2) and “add” them together. What is final (J,m_ℓ)?

(1) z-component is added: $m_\ell = m_1 + m_2$.

(2) $| J_1 - J_2| \leq J \leq J_1 + J_2$.

Parity:

Wave functions Ψ such that Ψ(-r) =Ψ (r) have even parity; wave functions such that Ψ (-r) = -Ψ (r) have odd parity. Parity is a quantum number and usually denoted by π. The parity of a single nucleon is: $\pi = (-1)^\ell$, the intrinsic parities of free nucleons are: $\pi_p = \pi_n = +1$

The Pauli Exclusion Principle: no two identical fermions can be in exactly the same quantum mechanical state.