1.The Electric Field

The electric field is said to exist in region of space around a charge object the source charge, when another charged object the test charge enters this electric field, electric force acts on it. As an example, consider Figure 1.2, which shows a small positive test charge $\mathbf{q_o}$ placed near a second object carrying a much greater positive charge Q.

Figure2.1

The electric field vector $\vec{\boldsymbol{E}}$ at a point in space is **defined** as the electric force $\mathbf{F_e}$ affect on a positive test charge $\mathbf{q_o}$ placed at that point divided by the test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_o} = \frac{N}{C} \quad in\ (SI)unit$$

Since $\mathbf{q_o}$ is always positive, the direction of $\boldsymbol{E}$ is the direction of $\boldsymbol{F}$. The (SI) unit of $\boldsymbol{E}$ is Newton per Coulomb (**N/C**). If the charge $\mathbf{q}$ is placed at a point with an electric field $\boldsymbol{E}$, this charge experiences an electric force given by:

$$\vec{F} = q\vec{E}$$

To determine the direction of an electric field, consider a point charge $\mathbf{q}$ as a source charge. This charge creates an electric field at all points in space surrounding it. A test charge $\mathbf{q_o}$ is placed at point **P**, a distance $\mathbf{r}$ from the source charge, as in Figure 2.2. where $\hat{\boldsymbol{r}}$ is a unit vector directed from $\mathbf{q}$ toward $\mathbf{q_o}$.

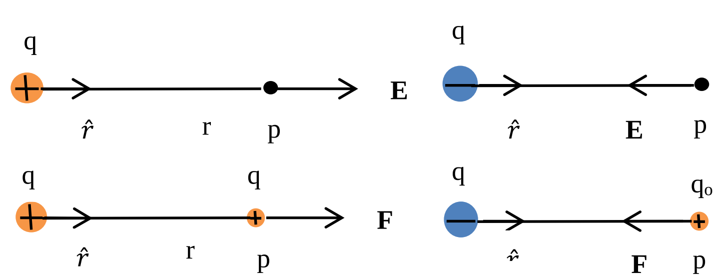


Figure 2.2

A. Electric Field of point charges

Let us calculate the electric field at a point (**P**). We assume existence of a test charge **q₀** at point (**P**) and point charge **q**, at distance **r** from it, see the figure. From coulomb's Law the force upon the test charge is

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{r^2}\hat{r}$$

Using the definition of **E** we get

$$\vec{E} = \frac{1}{4\pi\,\epsilon_o}\frac{q}{r^2}\hat{r}$$

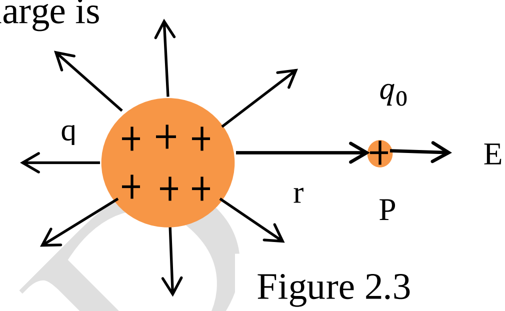


Figure 2.3

If the charge **q** is negative the direction of **E** will be reversed.

2. The electric field at point due to a group of point charges is the vector sum of the electric fields at that point due to each charge individually if we have n charges the net electric field **E** is:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + - - - + \vec{E}_n$$

where E_1, E_2 and E_n are the electric fields due to the charges q_1, q_2 and q_n respectively. The magnitude of the electric field is obtained as:

$$\Delta\vec{E} = \frac{1}{4\pi\,\epsilon_o}\frac{\Delta q}{r^2}\hat{r}$$

Example1: Two point charges of 7μC, -5 μC are located as shown in figure 2.4. Find the resultant of electric field at point P.

Solution: let us calculate the magnitudes of the electric field due to each charge.

$$\vec{E}_1 = 9\times10^9\frac{7\times10^{-6}}{(0.4)^2}\hat{j}$$

$(op)^2=(0.4)^2+(0.3)^2$

op=0.5m

$$\sin\theta = \frac{0.4}{0.5} = 0.8$$

$$\theta = 53°$$

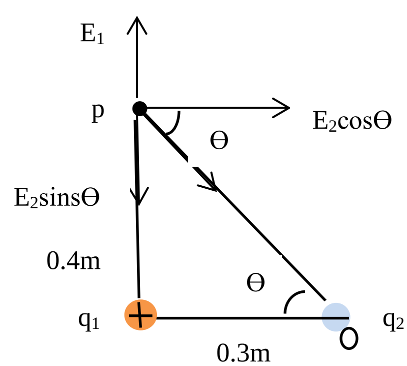


Figure 2.4

$$\vec{E}_2 = 9\times10^9\frac{5\times10^{-6}}{(0.5)^2}\,cos53°\hat{\imath} - 9\times10^9\frac{5\times10^{-6}}{(0.5)^2}\hat{j}$$

$$\vec{E}_{\mathrm{p}} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_{\mathrm{p}} = 9 \times 10^9 \frac{7 \times 10^{-6}}{(0.4)^2}\hat{\mathrm{j}} + 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} cos53°\hat{\imath} - 9 \times 10^9 \frac{5 \times 10^{-6}}{(0.5)^2} sin53°\hat{\jmath}$$

$$\vec{E}_{\mathrm{p}} = (2.5 \times 10^5 \hat{\jmath} + 1.1 \times 10^5 \hat{\imath}) N/C$$

B. Electric Field of A continuous charge distribution.

To evaluate the electric field of such a configuration, the following procedure are used:

1. We divide the distribution into small elements such as the electric field for each element can calculate easily.
2. Each element is assumed a charge **Δq** as shown in Figure 2.5.
3. Next we calculate the electric field due to one of these elements at the point in question.

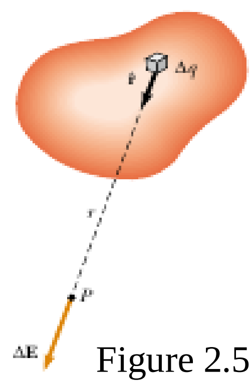


Figure 2.5

4. Finally we integrate over the charge distribution to evaluate the total electric field due to the whole charge distribution.

$$\mathrm{d}\vec{E} = \frac{1}{4\pi \in_{\mathrm{o}}} \frac{\mathrm{dq}}{\mathrm{r}^2}\hat{r}$$

The integration should be performed for each component of the electric field.

$$\Delta\vec{\mathrm{E}} = \frac{1}{4\pi \in_{\mathrm{o}}} \sum_i \frac{\Delta \mathrm{q}}{\mathrm{r}_i^2}\widehat{r_i}$$

When Δq_i is very small $\Delta q_i \to 0$ and the summation convert to integral

$$\sum_i \Delta\vec{\mathrm{E}} = \frac{1}{4\pi \in_{\mathrm{o}}} \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta \mathrm{q}_i}{\mathrm{r}_i^2}\widehat{r_i}$$

$$\int d\vec{E} = \mathrm{K} \int \frac{dq}{r^2}\widehat{r_i}$$

●. Charge density

when dealing with continuous charge distribution it is convenient to use the concept of the following:

1.If the charge is distribution a long a line we define the linear charge density $\boldsymbol{\lambda}$ as $\boldsymbol{\lambda} = \frac{\boldsymbol{dq}}{\boldsymbol{d\ell}}$

2.If the charge is distribution over a surface we define the surface charge density $\boldsymbol{\sigma}$ as $\boldsymbol{\sigma} = \frac{\boldsymbol{dq}}{\boldsymbol{d}\mathrm{A}}$

3.If the charge is distribution with a volume we define the volume charge density $\boldsymbol{\rho}$ as $\boldsymbol{\rho} = \frac{\boldsymbol{dq}}{\boldsymbol{d}\mathrm{v}}$

If the charge is uniformly distribution a long line, outer a surface or within a volume, the charge distribution are constant. In this case the total charge Q can be obtained by integrating both sides of the above equations obtaining:

$Q = \lambda\ell$, $Q = \sigma\mathrm{A}$, $Q = \rho\mathrm{v}$

C. Application on calculation electric field

1: A wire of length **L** lying along the x-axis is uniformly charged with a total charge **Q**. Calculate the electric field at point **p** along the x-axis of the wire at distance **D** from the end, as shown in the figure 6.2.

Solution: We divide the wire into small elements each of length dx and charge dq.

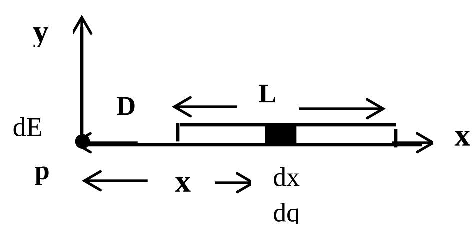


Figure 2.6

$$\mathrm{d}E = \mathrm{k}\frac{\mathrm{dq}}{\mathrm{x}^2}$$

$$\int dE = \mathrm{k}\int \frac{dq}{x^2}$$

Expressing dq in term of the charge density λ as $\boldsymbol{\lambda} = \frac{\boldsymbol{dq}}{\boldsymbol{dx}}$, $dq = \lambda\mathrm{dx}$

$$E = \mathrm{k}\lambda\int_D^{L+D} \frac{dx}{x^2}$$

$$E = \mathrm{k}\lambda - \frac{1}{x}\Big|_{\mathrm{D}}^{\mathrm{L+D}} = K\lambda[-\frac{1}{L+D} - \left(-\frac{1}{D}\right)]$$

$$E = \mathrm{k}\lambda - \frac{1}{x}\Big|_{\mathrm{D}}^{\mathrm{L+D}} = K\lambda[-\frac{1}{L+D} - \left(-\frac{1}{D}\right)] = K\lambda\frac{-D + D + L}{D(D+L)}$$

$$E = \mathrm{k}\lambda\frac{L}{D(D+L)}$$

If the **p** point is very far the D>>L and L is neglected and $Q = \lambda L$

$$\boldsymbol{E} = \mathbf{k}\frac{\boldsymbol{Q}}{\boldsymbol{D}^2}$$

2: A ring of radius **R** has a uniform charge distribution of magnitude **Q**. Calculate the electric field along the x-axis of the ring at point **p** laying a distance **D** from the center of the ring.

Solution: Symmetry considered: For every charge element dq considered, there exists $d\acute{q}$.

We divide the ring into small elements each of length dx and charge **dq**. The electric field due to one element is:$dx_2 d\bar{q}$

$$\mathrm{d}E = \mathrm{k}\frac{\mathrm{dq}}{\mathrm{r}^2}$$

Because the symmetry about the x-axis every charge element dq considered, there exists $d\acute{q}$. These elements produce an equal **d*E*** but different in direction as shown in the figure 2.7:

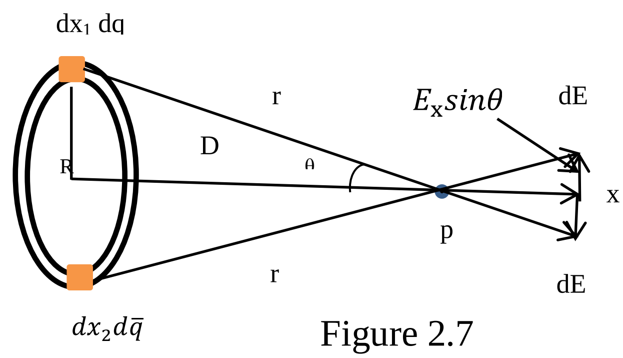


Figure 2.7

$$\mathrm{d}E_\mathrm{x} = \mathrm{dE}\cos\theta \qquad \cos\theta = \frac{\mathrm{D}}{\mathrm{r}}$$

$$r^2 = (R^2 + D^2) \qquad r = (R^2 + D^2)^{\frac{1}{2}}$$

$$\mathrm{d}E_\mathrm{x} = \mathrm{k}\frac{q}{r^2} \times \frac{D}{r} = \mathrm{k}\frac{qD}{(R^2+D^2)(R^2+D^2)^{\frac{1}{2}}}$$

$$\mathrm{d}E_\mathrm{x} = \mathrm{k}\frac{Ddq}{(R^2 + D^2)^{3/2}}$$

$$\int dE_\mathrm{x} = \mathrm{k}\frac{D}{(R^2 + D^2)^{3/2}}\int_0^Q dq$$

$$E_\mathrm{x} = \mathrm{k}\frac{DQ}{(R^2 + D^2)^{3/2}}$$

D>>R R is neglected R⇨0

$$E_\mathrm{x} = \mathrm{k}\frac{DQ}{(0 + D^2)^{3/2}} = \mathrm{k}\frac{DQ}{D^3}$$

$$E_x = k\frac{Q}{D^2}$$

3: A disk of radius **R** has a uniform charge density **σ** calculate the electric field along the x-axis of the disk at point **p** lying a distance **D** from the center of the disk.

Solution: We divided the disk into thin rings each of thickness **dr** and charge **dq**. The electric field due to one element with radius **r** is as shown in the figure 2.8:

$$dE = k\frac{Ddq}{(r^2+D^2)^{3/2}}$$

Integrating the last result we get

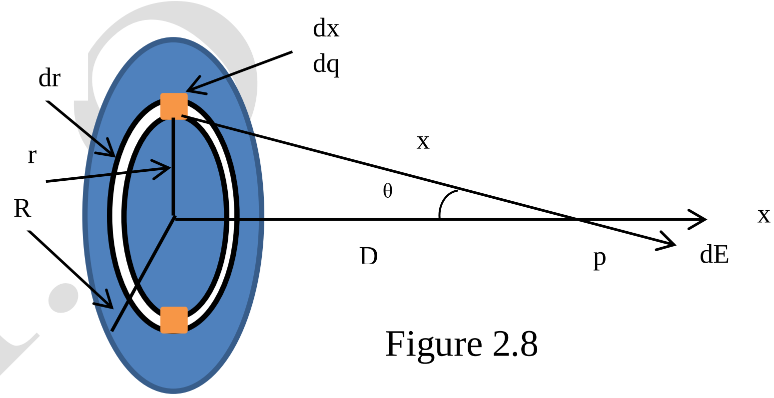


Figure 2.8

$$\int dE = kD\int\frac{dq}{(r^2+D^2)^{3/2}}$$

$A = \pi r^2$ *are of circle*

$$dq = \sigma dA = \sigma(2\pi r dr)$$

$$r^2 + D^2 = u$$

$$du = 2rdr$$

$$\int dE = kD\pi\sigma\int\frac{2rdr}{(r^2+D^2)^{3/2}}$$

$$\int\frac{du}{u^{3/2}} = \int u^{-3/2}du = \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}$$

$$E = kD\pi\sigma\frac{(D^2+r^2)^{-1/2}}{-\frac{1}{2}}\Big|_0^R = kD\pi\sigma 2\left[-\frac{1}{(D^2+r^2)^{\frac{1}{2}}}\right]\Big|_0^R$$

$$E = k\pi\sigma 2\left[-\frac{D}{(D^2+r^2)^{\frac{1}{2}}}\right]\Big|_0^R$$

$$E = 2k\pi\sigma\left[-\frac{D}{(D^2+R^2)^{\frac{1}{2}}} - (-\frac{D}{(D^2+0)^{\frac{1}{2}}})\right] = 2k\pi\sigma\left[\frac{D}{D} - \frac{D}{\sqrt{(D^2+R^2)}}\right]$$

$$E = 2k\pi\sigma\left[1 - \frac{D}{\sqrt{(D^2+R^2)}}\right]$$

When R $\rightarrow \infty$ or D=0 we get ⇨ $E = 2k\pi\sigma\left[1 - \frac{0}{\sqrt{(0+\infty)}}\right] = 2k\pi\sigma \times 1$

$$E = 2k\pi\sigma = \frac{2\pi\sigma}{4\pi \in_0} = \frac{\sigma}{2 \in_0}$$

4. Electric dipole: An electric dipole consists of a pair of **+q** and **−q** charge particles, which are separated by a distance **d=2r**. The magnitude of the electric dipole moment is $\mathbf{p} = \mathbf{qd}$. The direction of the electric dipole moment is along the direction from the negative charge to the positive charge. . The Q is mid-point of locations of **−q** and **+q** is called the center of the dipole as shown in the figure 2.9:

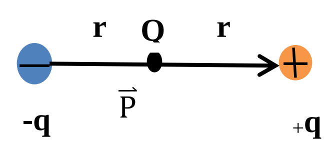


Figure 2.9

A. Electric field due to an electric dipole at an axil point.

The electric field at **p** due the positive charge is given by:

$$E_+ = \frac{1}{4\pi \in_o}\frac{q}{(r-d)^2}$$

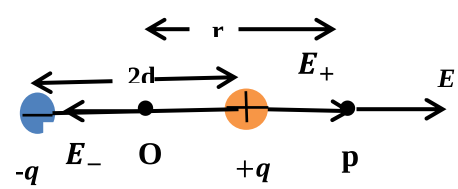


Figure 2.10

The electric field at p due the negative charge

$$E_- = \frac{1}{4\pi \in_o}\frac{-q}{(r+d)^2}$$

The electric field at point p is equal the vector sum of the electric fields at that point due to each charge:

$E = E_+ + E_-$

$$E = \frac{1}{4\pi \in_o}\left(\frac{q}{(r-d)^2} + \frac{-q}{(r+d)^2}\right) = \frac{q}{4\pi \in_o}\left[\frac{(r+d)^2-(r-d)^2}{((r-d)^2)^2}\right]$$

$$E = \frac{\text{q}}{4\pi \in_o}\left[\frac{r^2 + 2rd + d^2 - r^2 + 2rd - d^2}{((r-d)^2)^2}\right] = \frac{\text{q}}{4\pi \in_o}\frac{4rd}{((r-d)^2)^2}$$

$$E = \frac{1}{4\pi \in_o}\frac{2pr}{((r-d)^2)^2}\ , \qquad \text{p} = 2\text{qd}$$

If p is far the dipole, then r >> d ⇨ $d = 0$

$$E = \frac{1}{4\pi \in_o}\frac{2pr}{r^4} = \frac{1}{4\pi \in_o}\frac{2p}{r^3}$$

B. The electric field in the plane that bisects and is perpendicular to the dipole.

Solution: We resolve $\boldsymbol{E}_+$ and $\boldsymbol{E}_-$ into components along the x and y directions. The components of $\boldsymbol{E}_+$ and $\boldsymbol{E}_-$ cancel out. But the x components of $\boldsymbol{E}_+$ and $\boldsymbol{E}_-$ add to yield the resultant field $\boldsymbol{E}$ see figure 2.11.

$$E = E_+\cos\theta + E_-\cos\theta = 2E\cos\theta\ , \qquad E_+ = E_-$$

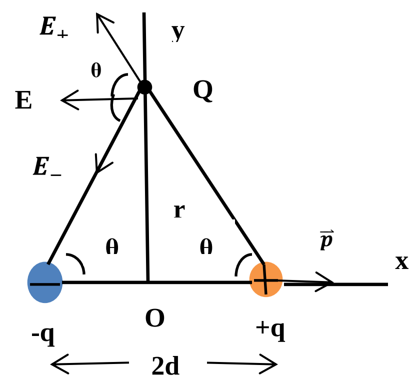


Figure 2.11

$$cos\theta = \frac{d}{(d^2 + r^2)^{1/2}}$$

$$(\text{الوتر})^2 = (d^2 + r^2)$$

$$E = 2 \times \frac{1}{4\pi \in_o}\frac{q}{(d^2 + r^2)} \times cos\theta$$

$$E = 2\frac{1}{4\pi \in_o}\frac{q}{(d^2 + r^2)}\frac{d}{(d^2 + r^2)^{1/2}} = \frac{2\text{qd}}{4\pi \in_o \ (d^2 + r^2)^{3/2}}$$

d is negligible compared to **r**

$$E = \frac{2\text{qd}}{4\pi \in_o \ (0 + r^2)^{3/2}} \qquad \text{p} = 2\text{qd}$$

$$E = \frac{1}{4\pi \in_o}\frac{p}{r^3}$$

Direction of $\boldsymbol{E}$ is opposite to that of dipole moment vector.

5. Lines of the Electric Force

The electric field in a region of space is represented by imaginary lines known as electric field force introduce by Faraday. The electric field lines have the following properties:

1.The electric field vector ***E*** is tangent to the electric field line at each point.

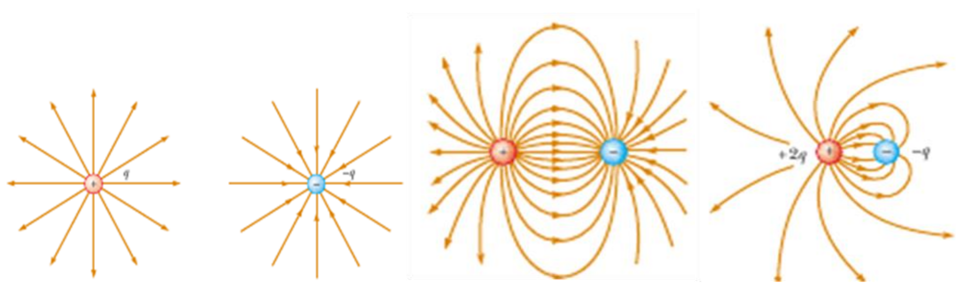
2. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

3.The lines must be begin at positive charges and terminate at negative charges.

4. No two field lines can crosses.

5.The number of lines drawn leaving appositive charge and approaching a negative is proportional to the magnitude of the charge.

The electric field lines associated with a positive charge **+2q** and a negative charge **-q**. In this case, the number of lines leaving **+ 2q** is twice the number terminating at **–q** see figure 2.12- d



a b c d

Figure 2.12

6. Motion of charged particles in a uniform Electric Field

The electric force obeys Newton's Law. If a particle of mass **m** and charge **q** exists in a uniform electric field **E** the electric force acting on the charge is

$$\vec{F_e} = q\vec{E} = m\vec{a}$$

Since $\boldsymbol{E}$ is uniform a constant and the formulas of motion gives with constant **a** acceleration.

$$\vec{a} = \frac{q\vec{E}}{m}$$

Example: A positive point charge **q** of mass **m** is released from rest in a uniform electric field see figure 13.2. Describe the motion?
Solution:

$$\vec{a} = \frac{q\vec{E}}{m} i$$

After a time **t** the particle is travelled a distance **x** is given by:

$$x = v_o t + \frac{1}{2} a t^2 , \qquad v_o = 0 \ at\ rest$$

$$x = \frac{1}{2}\frac{qE}{m} t^2$$

$$v^2 = v_o^2 + 2ax = 0 + 2\left(\frac{qE}{m}\right)\left(\frac{qE}{2m} t^2\right)$$

$$v = \frac{qEt}{m}$$

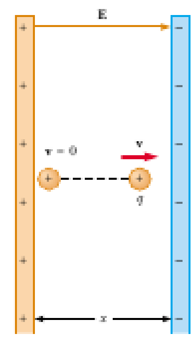


Figure 2.13

The third kinematic equation gives us:

$$v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right) x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$

$$E_K = \frac{1}{2} m v_f^2 = \frac{1}{2} m \left(\frac{2qE}{m}\right) \Delta x = qE\Delta x$$

$$E_K = F_e \Delta x = W$$

Example: An electron enters the region of a uniform electric field as shown in Figure, with $v_i = 3\times10^6$ m/s and **E**=200N/C. The horizontal length of the plates is ℓ =0.1m. 1. Find the acceleration of the electron while it is in the electric field, 2. If the electron enters the field at time **t** = 0, find the time at which it leaves the field, 3. If the vertical position of the electron as it enters the field is y_i = 0, what is its vertical position when it leaves the field see figure 2.14?

Solution: The electron attraction with (+ plate), $\vec{F_e} = -eE$ is downward and so F_y is negative and the electron's acceleration is constant:

$$\vec{a} = -\frac{eE}{m_e}\vec{j}$$

$$\vec{a} = -\frac{(1.6\times10^{-19}C\times200N/C)}{1.67\times10^{-31}kg}\vec{j}$$

$$\vec{a} = -3.5\times10^{-13}\,\vec{j}\,m/sec^2$$

$$t = \frac{\ell}{v_i} = \frac{0.1m}{3\times10^6 m/sec^2} = 3.33\times10^{-8} sec$$

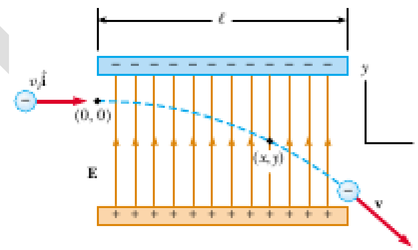


Figure 2.14

$$y_f = \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.5\times10^{-13}m/sec^2)(3.33\times10^{-8})^2 sec^2$$

$$y_f = 0.0195m$$

7. Electric Flux: the total number of the lines of force penetrating the surface is proportional to the product ***E*A**. This product of the magnitude of the electric field ***E*** and surface area **A** perpendicular to the field is called the electric flux $\boldsymbol{\varphi_E}$ see figure 2.15.

1. $\boldsymbol{\varphi_E = EA}$

From the SI units of **E** and **A**, $\boldsymbol{\varphi_E}$ is scalar quantity has (N.m^2/C).

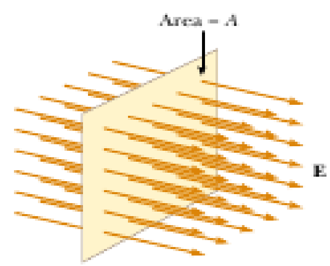


Figure 2.15

2.If the surface under consideration is not perpendicular to the field see figure 2.16, the flux through it must be less than that given by Equation $\varphi_E = EA$

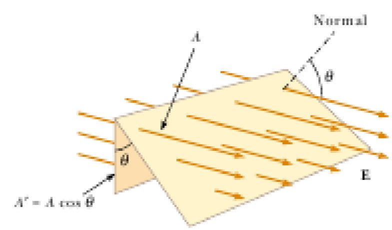


Figure 2.16

Consider the closed surface in Figure 2.17.

The vectors A_i point in different directions for the various surface elements, but at each point they are normal to the surface and, by convention, always point outward. At the element labeled(1), the field lines are crossing the surface from the inside to the outside and $\theta < 90°$; hence, the $\Delta E_{\varphi} = E \Delta A_1$ through this element is positive. For element(2), the field lines perpendicular to the vector ΔA_2; thus, $\theta = 90°$ and the flux is zero. For elements such as (3), where the field lines are crossing the surface from outside to inside $180° > \theta > 90°$, and the flux is negative because cosθ is negative.

The net flux through the surface is proportional to the net number of lines leaving the surface, where the net number means the number leaving the surface minus the number entering the surface.

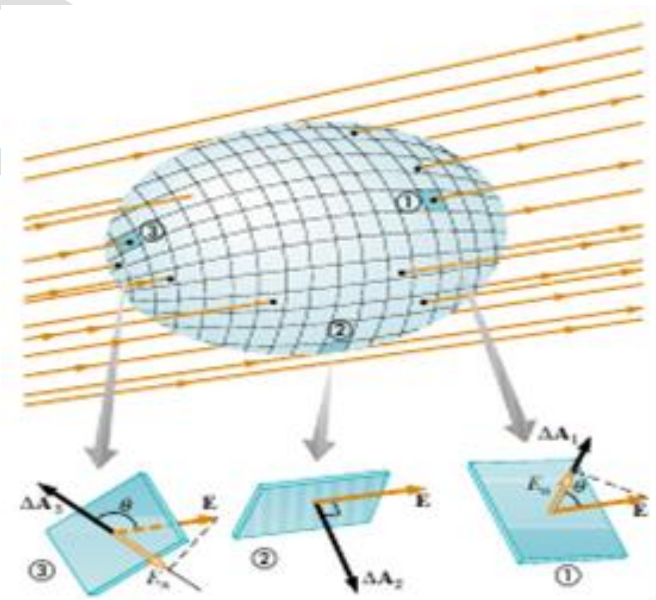


Figure 17.2

If more lines are entering than leaving, the net flux is negative. Using the symbol $\oint$, to represent an integral over a closed surface, we can write the net flux E_{φ} through a closed

$$\varphi_E = \oint E.dA = \oint E_n \, dA$$

Where E_n represents the component of the electric field normal to the surface. If the field is normal to the surface at each point and constant in magnitude.

Example: Flux through a Cube

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edge length $\boldsymbol{\ell}$, oriented as shown in figure 2.18.

Solution: The net flux is the sum of the fluxes through all faces of the cube. The flux through faces (**3, 4, 5, 6**) is zero because ***E*** is perpendicular to ***dA***

The net flux through faces 1 and 2 is

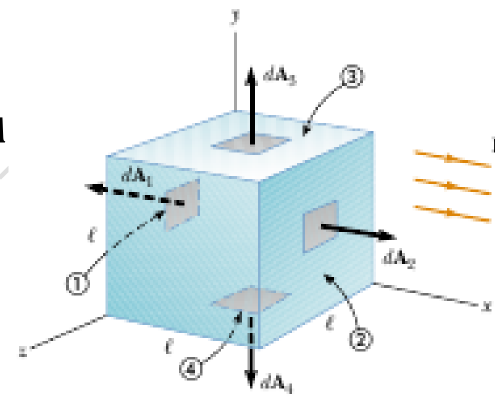


Figure 2.18

$$\varphi_E = \oint E.dA_1 + \oint E.dA_2$$

$$\varphi_E = \int EdA\,cos180° + \int EdA\cos 0°$$

$$\varphi_E = -E\ell^{\mathbf{2}} + E\ell^2 = 0$$

Therefore, the net flux over all six faces is

$$\varphi_E = 0$$

8. GAUSS'S L AW

The electric flux through any closed surface is equal to the net q inside the surface divided by $\in_o$ that is

$$\varphi_E = \oint E.dA = \frac{q}{\in_o}$$

1.Where the charge **q** inside the surface and the circle the integral sign indic ates that the integration is over a closed surface.

Let us briefly Gauss's Law by considering a positive point charge **q** surrounded by two closed surface **S$_1$** is spherical, whereas **S$_2$** and **S$_3$** is irregular see figure 19.2. Coulomb's Law tell us that the magnitude of the electric field is constant everywhere on the spherical surface and given as

$$\vec{E} = \mathrm{K}\frac{|\mathrm{q}|}{\mathrm{r}^2}\,\hat{r}$$

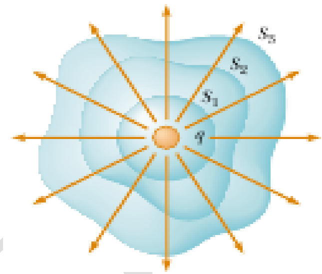


Figure 2.19

Since the electric field direction is radial,
we can evaluate the flux through **S_1** as:

$$\varphi_E = \oint EdA = EA$$

$$\varphi_E = \frac{1}{4\pi \in_o}\frac{q}{r^2}(4\pi r^2) = \frac{q}{\in_o}$$

The number of the field lines crossing **S_1** is the same as that lines crossing **S_2**, that is the flux through the two surfaces are equal and independent of their shape.

2.If the charge exists outside a closed surface the electric field line entering the surface must leave that surface. Hence the electric flux through that the surface is zero see the figure 20.2 .

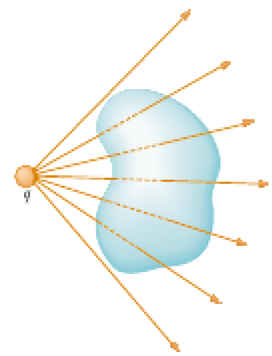


Figure 20.2

For this way to be as easy as possible we must be able to choose a hypothetical closed surface (Gaussian surface) such that the electric over its surface is constant.

This can be attained if the following remarks are satisfied.

1.The charge distribution must have a high degree of symmetry (spherical, cylindrical, with infinite length and plan with infinite extends).

2. The Gaussian surface should have the same symmetry as that of the charge distribution.

3. The point at which ***E*** is to be evaluated should lie on the Gaussian surface.

4. If $\boldsymbol{E}$ is parallel to the surface or zero at every point then $\varphi = \oint E.dA = 0$.

5. If $\boldsymbol{E}$ is perpendicular to the surface at every point, and since E is constant then:

$$\boldsymbol{\varphi = \oint E.dA = EA = \frac{q}{\in o}}$$

A.Applications of Gauss's Law

Example 1: Find the electric field a distance **r** from a point charge **q**.

Solution: Choose the Gaussian surface as sphere of radius **r** see figure 2.21.

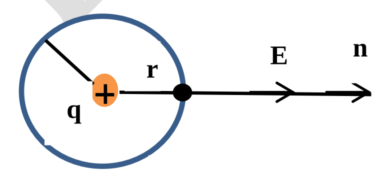


Figure 2.21

$$\varphi_E = \oint E.dA = \frac{q}{\epsilon_o}$$

$$\varphi = E(4\pi r^2) = \frac{q}{\epsilon_o}$$

$$E = \frac{q}{4\pi\epsilon_o r^2} = K\frac{q}{r^2}$$

We clear that $\boldsymbol{E}$ and **dA** are parallel and $\boldsymbol{E}$ is constant over the surface.

Example 2: Find the electric field a distance **r** from an infinite line charge of uniform density **λ**.

Solution: Gaussian surface are select a circular cylinder of radius (**r**) with high (**h**) and coaxial with the line charge see figure 2.22.

The cylinder has three surfaces; the integral in Gauss's Law has to be split into three parts.

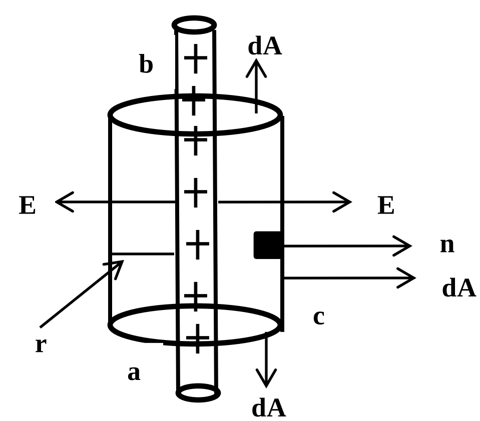


Figure 2.22

$$\varphi_E = \oint E.dA = \frac{q}{\epsilon_o}$$

$$\oint_a E.dA\cos 90° + \oint_b E.dA\cos 90° + \oint_c E.dA\cos 0° = \frac{q}{\epsilon_o}$$

$$\varphi_E = \oint E.dA = \frac{q}{\epsilon_o}$$

For the symmetry of the system, E is parallel to both bases furthermore it has a constant magnitude and directed radially outward at every point on the curved surface of the cylinder.

$$EA = E(2\pi rh) = \frac{q}{\epsilon_o}$$

$$q = \lambda h$$

$$E = \frac{\lambda}{2\pi r\epsilon_o}$$

If the wire is not too long it's the ends will be closed to any Gaussian surface. Since the electric field at closed to the ends is not uniform it will impossible to many the integral of Gauss's Law.

Example3: Find the electric field ***E*** due to non-conductor infinite plane with uniform surface charge density **σ**.

Solution: We select as a Gaussian surface a small cylinder whose axis is perpendicular to the plane and whose ends have an area **A** see figure 2.23.

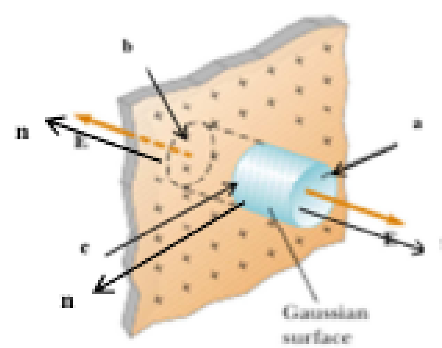
$$\oint_a E.dA\cos 0° + \oint_b E.dA\cos 0° + \oint_c E.dA\cos 90° = \frac{q}{\epsilon_o}$$

$$EA + EA = \frac{q}{\epsilon_o}$$

$$q = \sigma A$$

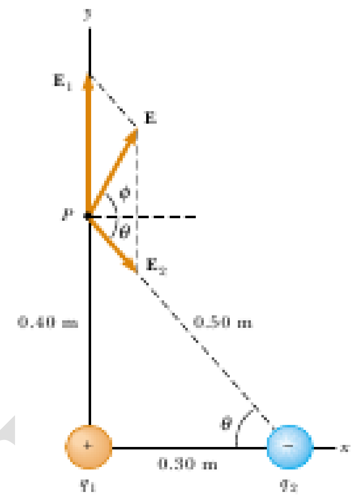
$$2EA = \frac{\sigma A}{\epsilon_o}$$

$$E = \frac{\sigma}{2\epsilon_o}$$

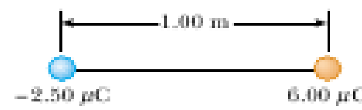


Questions

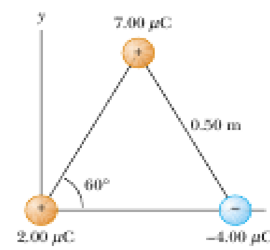
Q1: A charge $q_1 = 7\mu C$ is located at the origin, and a second charge $q_2 = -5\mu C$ is located on the x axis, 0.3 m from the origin (as shown in Figure). Find the electric field at the point P, which has coordinates (0, 0.4) m.



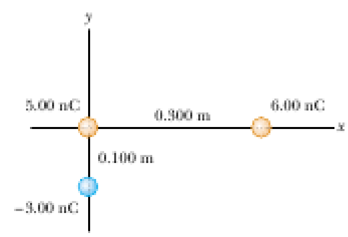
Q2: In Figure, determine the point (other than infinity) at which the electric field is zero.



Q3: Three charges are at the corners of an equilateral triangle as shown in Figure.
(a) Calculate the electric field at the position of the $2\mu C$ charge due to the $7\mu C$ and $-4\mu C$ charges.
(b) Use your answer to part (a) to determine the force on the $2\mu C$ charge.

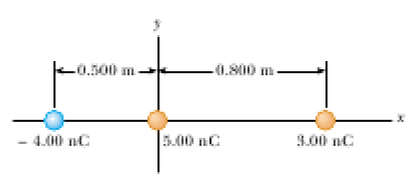


Q4: Three point charges are arranged as shown in Figure. (a) Find the vector electric field that the $6\mu C$ and $3\mu C$ charges together create at the origin. (b) Find the vector force on the $5\mu C$ charge.



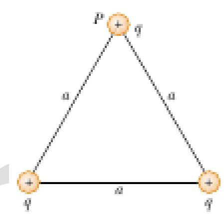
Q5: Two $2\mu C$ point charges are located on the x axis. One is at x=1m, and the other is at x=-1m. (a) Determine the electric field on the y-axis at y= 0.5m. (b) Calculate the electric force on a $-3\mu C$ charge placed on the y-axis at y=0.5m.

Q6: Three point charges are aligned along the x axis as shown in Figure. Find the electric field at (a) the position (2, 0) and (b) the position (0, 2).



Q7: A uniformly charged disk of radius 35cm carries charges with a density of 7.9×10^{3} C/m^2. Calculate the electric field on the axis of the disk at (a) 5cm, (b) 10cm, (c) 50cm, and (d) 200 cm from the center of the disk.

Q8: Three equal positive charges q are at the corners of an equilateral triangle of side a as shown in Figure. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?



Q9: A uniformly charged ring of radius 10cm has a total charge of 75μC. Find the electric field on the axis of the ring at (a) 1cm, (b) 5cm, (c) 30cm, and (d) 100 cm from the center of the ring.

Q10: A proton accelerates from rest in a uniform electric field of 640 N/C. At some later time, its speed is $1.2\times10^{6}\ {}^{m}/_{sec}$ (nonrelativistic, because v is much less than the speed of light). (a) Find the acceleration of the proton. (b) How long does it take the proton to reach this speed? (c) How far has it moved in this time? (d) What is its kinetic energy at this time?