and after substituting (13.8) into (13.9)

$$\frac{d^{2\prime} \boldsymbol{r}}{dt^2} = \frac{d}{dt}\left(\frac{d\boldsymbol{r}}{dt} - [\boldsymbol{r} \times \boldsymbol{\Omega}]\right) - \left[\left(\frac{d\boldsymbol{r}}{dt} - [\boldsymbol{r} \times \boldsymbol{\Omega}]\right) \times \boldsymbol{\Omega}\right] \tag{13.10}$$

$$= \frac{d^2\boldsymbol{r}}{dt^2} - 2\left[\frac{d\boldsymbol{r}}{dt} \times \boldsymbol{\Omega}\right] + \left[[\boldsymbol{r} \times \boldsymbol{\Omega}] \times \Omega\right]. \tag{13.11}$$

Eq. (13.11) states that the acceleration in an inertial system equals the acceleration relative to the rotating system plus the Coriolis acceleration (second term) plus the centripetal acceleration (third term).

Vector notation of the Coriolis force and acceleration, respectively:

$$\boldsymbol{F}_c = -2m[\boldsymbol{\Omega} \times \boldsymbol{v}] \tag{13.12}$$

$$\boldsymbol{a}_c = -2[\boldsymbol{\Omega} \times \boldsymbol{v}] \tag{13.13}$$

where $\boldsymbol{\Omega} = (0, \Omega\cos\phi, \Omega\sin\phi)$ is the vector of the earth's rotation ($|\Omega| = 7.29 \cdot 10^{-5}$ rad s^{-1}).

In its cartesian components, the Coriolis acceleration is:

$$\begin{aligned} \boldsymbol{a}_c = {} & 2(\Omega v \sin\phi - 2\Omega w \cos\phi)\ \boldsymbol{i} \\ & - 2\Omega u \sin\phi\ \boldsymbol{j} \\ & + 2\Omega u \cos\phi\ \boldsymbol{k}, \end{aligned}$$

Usually, the vertical component of the Coriolis force is small compared to the gravitational force and vertical wind speeds are much smaller than horizontal ones, so that these terms can be neglected leaving only horizontal components:

$$\begin{aligned} \boldsymbol{a}_c \approx \boldsymbol{a}_{c,h} &= -2\Omega\sin\phi[\boldsymbol{k} \times \boldsymbol{v}_h] \\ &= -f[\boldsymbol{k} \times \boldsymbol{v}_h], \end{aligned} \tag{13.14}$$

where $f = 2\Omega\sin\phi$ ($\sim 10^{-4}\ s^{-1}$ in midlatitudes) is the *Coriolis parameter*. From Eq. 13.14 it is clear, that the Coriolis force vanishes at the equator and is perpendicular to the horizontal wind vector $\boldsymbol{v}_h$.

13.2.3 Friction Force

The air motion is slowed down at the earth's surface due to the friction force acting in the lowest layers of the atmosphere ($z < \sim 1000\ m$). For $z = 0$, it is $\boldsymbol{v}_h = 0$. The friction force is determined by

- wind velocity
- surface roughness
- thermal stability of the atmosphere.

A simple relationship for the friction force is:

$$\boldsymbol{F}_{fr,h} = -m\mu\boldsymbol{v}_h; \qquad \boldsymbol{a}_{fr,h} = -\mu\boldsymbol{v}_h \tag{13.15}$$

with the friction (or viscous) coefficient μ.
The friction force is directed opposed to the wind flow.

The friction force is mentioned here only to complete the list of forces acting on an air parcel. Its detailed description is out of the scope of this text, but is of major importance for the description of turbulent flows in the atmospheric boundary layer.

13.3 Equation of Motion

Horizontal motions of fluid elements in the atmosphere are governed by the acting forces introduced in section 13.2:

$$\boldsymbol{F}_h = \boldsymbol{F}_{p,h} + \boldsymbol{F}_{c,h} + \boldsymbol{F}_{fr,h}. \tag{13.16}$$

The individual acceleration of fluid elements (air parcels) thus is:

$$\frac{d\boldsymbol{v}_h}{dt} = \boldsymbol{a}_h = \boldsymbol{a}_{p,h} + \boldsymbol{a}_{c,h} + \boldsymbol{a}_{fr,h}. \tag{13.17}$$

Inserting from Eq. (13.7), (13.14) and (13.15), the *horizontal equation of motion* can be written:

$$\frac{d\boldsymbol{v}_h}{dt} = -\frac{1}{\rho}\boldsymbol{\nabla_h} p - f(\boldsymbol{k} \times \boldsymbol{v}_h) - \mu \boldsymbol{v}_h \tag{13.18}$$

Note: Accelaration is due to a change in velocity of the motion as well as due to a change in direction of the motion.

13.4 Balances of the Horizontal Wind Field

13.4.1 Geostrophic Balance

Assuming a parcel of fluid in a height above the surface, where it does not experience any influence of the underlying surface on its flow, i.e., no friction force is acting on it. Comparing the typical magnitudes of the acceleration term and of the acting forces:

- acceleration: 10^{-4} ms^{-1},
- forces: 10^{-3} ms^{-1}

In good approximation, the acceleration can be neglected in most situations and an equilibrium between the horizontal pressure gradient force and the Coriolis force is established:

$$-\frac{1}{\rho}\boldsymbol{\nabla_h} p = f(\boldsymbol{k} \times \boldsymbol{v}_h) \tag{13.19}$$

or

$$\boldsymbol{v}_h = -\frac{1}{\rho f}\boldsymbol{\nabla_h} p \times \boldsymbol{k}. \tag{13.20}$$

The wind speed $\boldsymbol{v}_h$ representing this balance is called the *geostrophic wind* v_g and is written in absolute terms as

$$v_g = -\frac{1}{\rho f}\nabla_h p. \tag{13.21}$$

Fig. 13.2 shows that $\boldsymbol{v}_g$ is perpendicular to the pressure gradient $\boldsymbol{\nabla_h} p$ and parallel to the isolines of constant pressure (isobars). $\boldsymbol{v}_g$ is proportional to the distance between the isolines.

The geostrophic wind is a good measure for the general wind flow for a given site (area) because of its independence on specific surface characteristics (orography, obstacles, roughness).

Note: The geostrophic approximation is not valid in equatorial regions since the Coriolis force is not defined for $\phi = 0$.

The geostrophic assumption requires the abscence of horizontal accelerations:
$\longrightarrow$ straight steady flow (uncurved flow)
$\longrightarrow$ parallel to isolines of constant pressure
$\longrightarrow$ $v \sim$ distance between isobars

13.4.2 Friction Wind

In the atmospheric boundary layer (ABL) the friction force adds to the acting forces. The balance now is (Fig. 13.3):

$$-\frac{1}{\rho}\boldsymbol{\nabla_h} p - f(\boldsymbol{k} \times \boldsymbol{v}_h) + \mu \boldsymbol{v}_h = 0 \tag{13.22}$$

With decreasing wind velocity the Coriolis force also decreases. To compensate for the pressure gradient force, the wind has to turn left. This indicates that in the

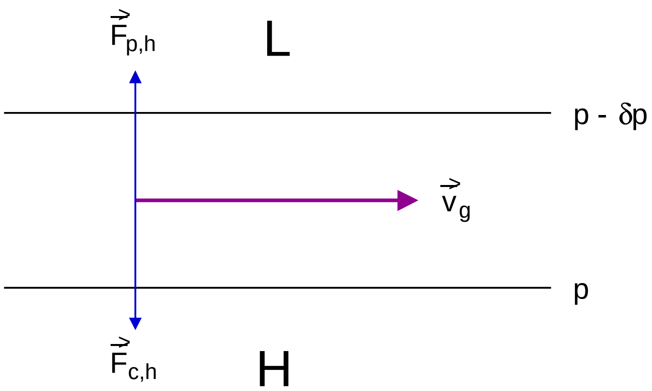

Fig. 13.2. The geostrophic balance. The horizontal components of the pressure gradient force $\boldsymbol{F}_{p,h}$ and the Coriolis force $\boldsymbol{F}_{c,h}$ are balanced. v_g is the geostrophic wind.

atmospheric boundary layer the horizontal wind has a component directed toward lower pressure (Fig. 13.4).

The deflection angle α depends on

– surface roughness
– latitude
– turbulence.

Typical values for the surface wind deflection are 15-30° over sea and 25-50° over land. The corresponding values for v/v_g are 0.8 and 0.5, respectively. Due to the decreasing Coriolis force α increases with decreasing latitude.

The vertical change of wind speed and direction due to the decreasing influence of friction with height is represented by the logarithmic Ekman spiral (Fig. 13.5).

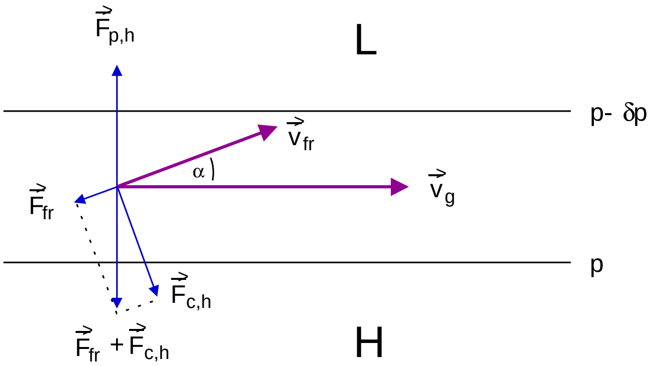

Fig. 13.3. The balance of horizontal forces in the atmospheric boundary layer. The pressure gradient force $\boldsymbol{F}_{p,h}$ is balanced by the sum of the Coriolis force $\boldsymbol{F}_{c,h}$ and the friction force $\boldsymbol{F}_{fr}$.

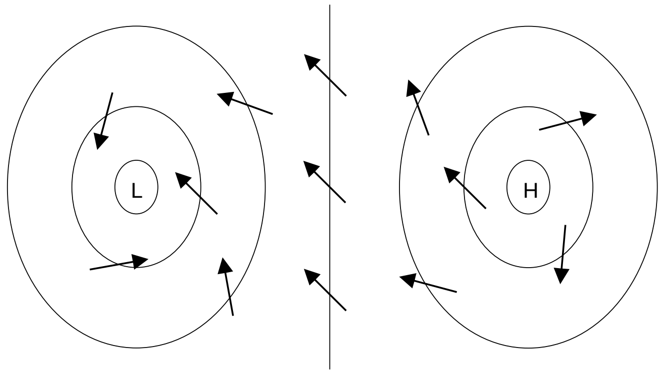

Fig. 13.4. Schematic surface wind pattern associated with centers of high and low pressure. The wind field is indicated by arrows, isobars are shown by solid lines. Note that this holds for the Northern hemisphere only. The circulation in the Southern hemisphere is clockwise for lows and counterclockwise for highs.

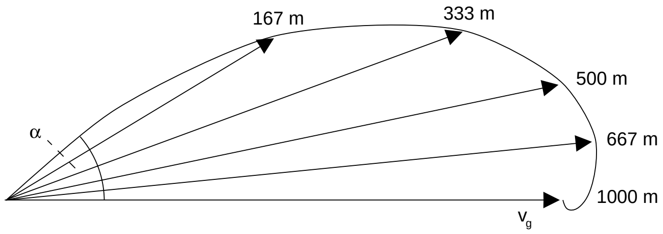

Fig. 13.5. The theoretical Ekman spiral describing the height dependence of the departure of the wind field in the boundary layer from geostrophic balance. α is the directional departure from the geostrophic wind.