Lecture (3)

Series Expansion Methods

3.1 Introduction

- The approximate solutions of partial differential equations (PDE's), such as advection-dispersion equations, can be found by Finite Difference methods, Series Expansion methods, or Finite Volume methods.
- The purpose of using approximation is to reduce the space of solution of every continuous differential equation from an indefinite number to a finite number of space or time nodes in order to increase the speed of computations.
- In the finite difference method, we replace each continuous differential operator (d) by discrete difference analog (Δ). This analog is an approximation written in terms of finite number of values of a variable at each time or space node.
- If the west-east scalar velocity, u_x , is a spatially continuous function at a certain time, then its value can be described as an west-east discretized grid (figure 3.1).

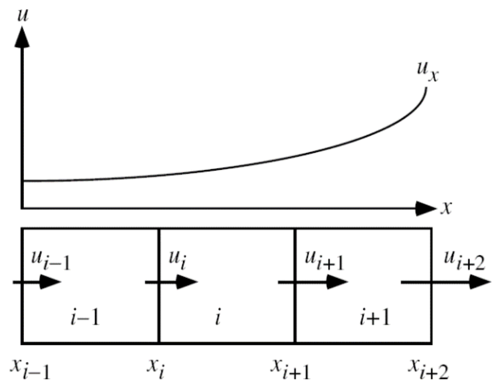

Fig.(3.1) Discretization of a continuous west-east scalar velocity u_x

- While in the series expansion method, the dependent variable in the PDE (such as u, v, w, or N) is replaced by a finite series approximates its value.
- For example, if the PDE of the west – east equation at the nude *i* for the concentration variable is

$$\frac{\partial N_i}{\partial t} + \frac{\partial (uN)_i}{\partial x} = 0 \qquad (1)$$

then the series expansion approximation for N at *i* :

$$N_i \approx N_i(x) = \sum_j N_j e_j(x) \qquad (2)$$

where $N_i(x)$ is a trial function. Now suppose that u is a constant value. The number of nodes j over which the trail function is approximated is called the trial space. The trail function is the summation, over each node in the trail space, for the true concentration, N_j, multiplied by a *basis function* $e_j(x)$. The difference between equation (1) when $N_i(x)$

is used and equation (2) when $\sum_j N_j e_j(x)$ is used, is the *residual* $R_i(x)$. The residual is the difference between the approximate function and the exact function.

The series expansion method which uses a *local* basis function is the finite-element method. The series expansion method which uses a *global* basis function counterpart to residual is the spectral method. The familiar series expansion method is the spectral method. The familiar finite element is the Galerkin finite- element method. With this method, the local basis function is also counterpart to the residual. The basis functions for the other finite element methods may or may not counterpart to the residual.

3.3 Spectral methods

- Spectral methods are powerful methods used for the solution of partial differential equations. Unlike finite difference methods, spectral methods are global methods, where the computation at any given point depends not only on information at neighboring points, but on information from the entire domain. Spectral methods converged exponentially, which makes them more accurate than local methods. Global methods are preferable to local methods when the solution varies considerably in time or space, when very high spatial resolution is required, and also when long time integration is needed.
- However, using finite difference methods to approximate solutions containing very significant spatial or temporal variation requires a very fine grid in order to accurately resolve the function. Clearly, the use of fine grids requires significant computational resources in simulations of interest to science and engineering. In the face of such limitations, we seek alternative schemes that will allow coarser grids, and therefore fewer computational resources. Spectral methods are such methods; they use all available function values to construct the necessary approximations.

3.4 Finite Element Methods

- The method was developed in the 1950s and originated from the need to solve complex elasticity and structural analysis problems in engineering. Mesh discretization of a continuous domain converted into a set of discrete sub-domains, usually called elements. The FE method is a numerical method to solve differential equations by discretizing the domain into a finite mesh. Numerically speaking, a set of differential equations are converted into a set of algebraic equations to be solved for unknown at the nodes of the mesh.
- The advantages of this method can be summarized as follows:

1. Numerical efficiency. 2. Treatment of nonlinearities. 3. Complex geometry: By the use of the FE method, any complex domain can be discretized by triangular elements in 2D and by tetrahedra elements in 3D. 4. Applicable to many field problems: The FE method is suited for structural analysis, heat transfer, fluid and acoustic analysis, etc.