where τ_{xz} represents vertical transport of horizontal momentum and K is the eddy viscosity. This assumption leads to the logarithmic wind profile:

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz} \tag{15.3}$$

where $k \simeq 0.4$ is the von Kármán constant and $u_* = \sqrt{\tau/\rho}$ is the friction velocity, a quantity characterizing the vertical turbulent flux of momentum.

To derive Eq. (15.3), it is used that K can be described in terms of the vertical wind speed gradient and the mixing length l, which represents the length scale necessary for changing the turbulent characteristics of a vertically moving eddy. l also characterizes the size of the eddies involved:

$$K = l^2 \, \frac{\partial u}{\partial z}, \tag{15.4}$$

where $l = k\,z$.

In its integrated form the logarithmic profile can be written as

$$u(z) = \frac{u_*}{k} \, ln(\frac{z}{z_0}), \tag{15.5}$$

where z_0 is an integration constant denoting the height where $u(z)$ = 0. z_0 can not directly be interpreted as a physical length, but depends on the size and structure of the local roughness elements. For this reason z_0 is called the roughness length. z_0 can experimentally be derived from measured wind profiles, but usually characteristic values for given surface types are used (Tab. 15.2.1).

Table 15.1. Typical values for the roughness length z_o in m.

ice	10^{-5}	high grass	0.04
smooth sea	2-3×10^{-4}	wheat	0.05
sand	10^{-4} - 10^{-3}	low woods	0.05 - 0.1
grassy surface	0.015 - 0.02	high woods	0.2 - 1.0
low grass, steppe	0.03	suburbia	1 - 2
flat country	0.02	city	1 - 5

Eq. (15.3) and (15.5) are valid only under several assumptions:
- only for mean values (periods longer than 10 min.)
- horizontally homogeneous surface
- neutral stability of the atmosphere

Given a wind speed value in height z_1 and the local roughness length z_0 the complete vertical profile can be calculated using Eq. (15.5):

$$\frac{u(z_1)}{u(z_2)} = \frac{ln(z_1/z_0)}{ln(z_2)(z_0)}. \tag{15.6}$$

Frequently, a simpler power law is used to describe the vertical wind profile:

$$\frac{u(z_1)}{u(z_2)} = (\frac{z_1}{z_2})^{\alpha}, \tag{15.7}$$

where α represents the influence of height, surface roughness and thermal atmospheric stability. An often used value is $\alpha = 1/7$. Due to the strong variability of α the use of the power law is not recommended.

For short period wind speed data ($<$ one hour, instantaneous) the actual atmospheric state (concerning stability) has to be taken into consideration. A commonly used method is the use of the boundary layer similarity theory (Monin-Obukhov theory) by applying a correction function to the logarithmic wind profile:

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}\, \phi(\frac{z}{L}), \tag{15.8}$$

where $\phi(z/L)$ is a universal function of the height z relative to the similarity scale L (Monin-Obukhov-length). For neutral atmospheres $\phi(z/L) = 1$.

15.2.2 The Ekman Layer

The layer on top of the surface layer characterized by increasing shear stress with height and a changing wind direction with height is called the Ekman layer.

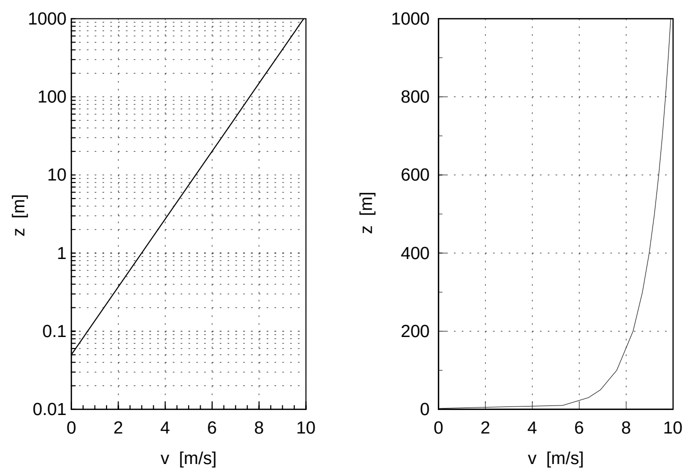

Fig. 15.2. The logarithmic wind profile in logarithmic (left) and linear right) height scale (z_0 = 0.05 m, u^* = 0.4 ms^{-1}).