Mustansiriyah University / College of Science
Department of Physics

optics

المحاضرة (8)

المرحلة: الثالثة

الفصل الدراسي الثاني

Wave Optics

اعداد:

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الجامعة المستنصرية /كلية العلوم/قسم الفيزياء

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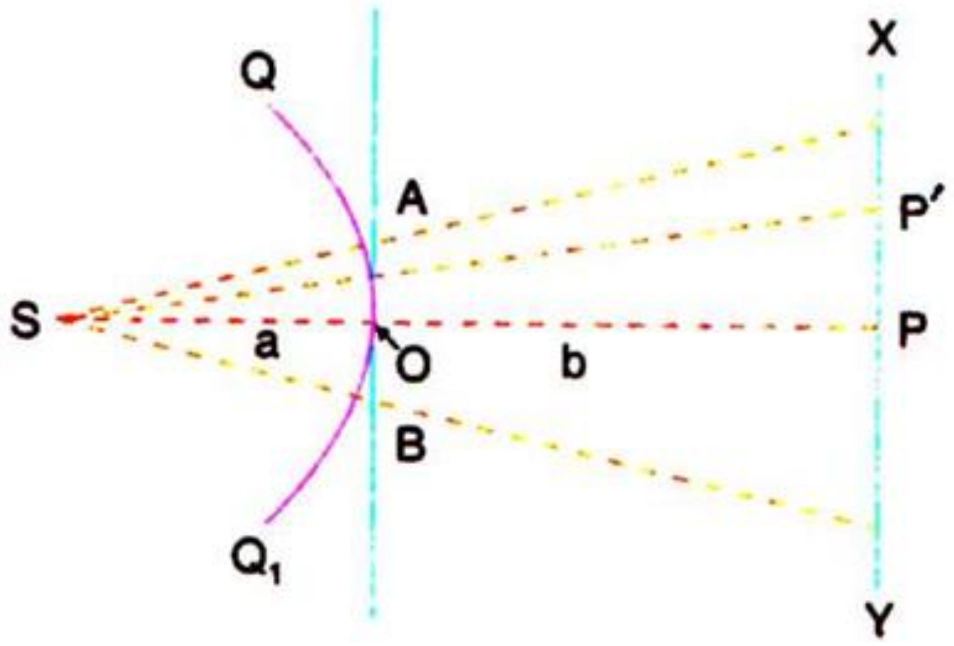
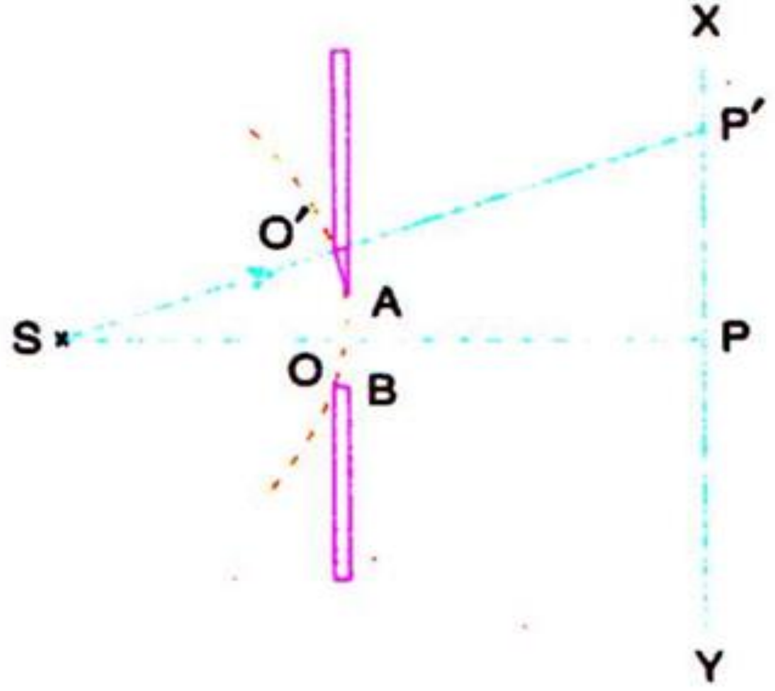
Diffraction at a Circular Aperture:





Mathematical Treatment of Diffraction at a Circular Aperture:

Let δ be the path difference for the wave reaching P along the paths SAP and SOP.

$$SO = a \quad ; \quad OP = b \quad ; \quad OA = r$$

$$\delta = SA + AP - SOP$$

$$\delta = (a^2 + r^2)^{1/2} + (b^2 + r^2)^{1/2} - (a + b)$$

$$\delta = a\left(1 + \frac{r^2}{a^2}\right)^{1/2} + b\left(1 + \frac{r^2}{b^2}\right)^{1/2} - (a + b)$$

$$\delta = a\left(1 + \frac{r^2}{2a^2}\right) + b\left(1 + \frac{r^2}{2b^2}\right) - (a + b)$$

$$\delta = \left(a + \frac{r^2}{2a}\right) + \left(b + \frac{r^2}{2b}\right) - (a + b)$$

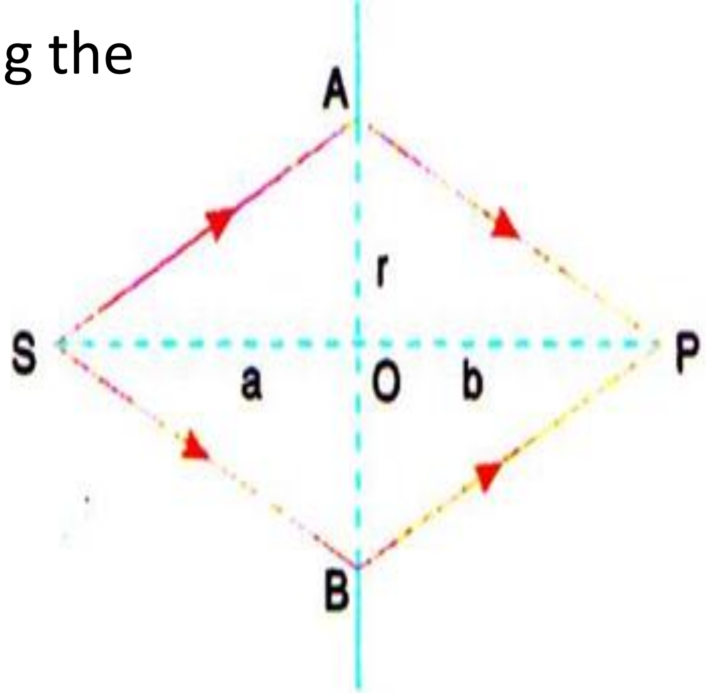
$$\delta = \frac{r^2}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\frac{2\delta}{r^2} = \left(\frac{1}{a} + \frac{1}{b}\right)$$

$$\delta = \frac{n\lambda}{2} \text{ or } 2\delta = n\lambda$$

$$\frac{1}{a} + \frac{1}{b} = \frac{n\lambda}{r^2}$$

$$\frac{1}{b} = \frac{1}{f} = \frac{n\lambda}{r^2}$$



Intensity at a Point Away From the Centre:

$$\delta = BP' - AP'$$

$$\delta = \sqrt{b^2 + (x+r)^2} - \sqrt{b^2 + (x-r)^2}$$

$$\delta = b\left(1 + \frac{(x+r)^2}{2b^2}\right) - b\left(1 + \frac{(x-r)^2}{2b^2}\right)$$

$$\delta = \left(b + \frac{(x+r)^2}{2b}\right) - b\left(1 + \frac{(x-r)^2}{2b^2}\right)$$

$$\delta = \frac{1}{2b}\left((x+r)^2 - (x-r)^2\right)$$

$$\delta = \frac{1}{2b}(4xr) = \frac{2rx}{b}$$

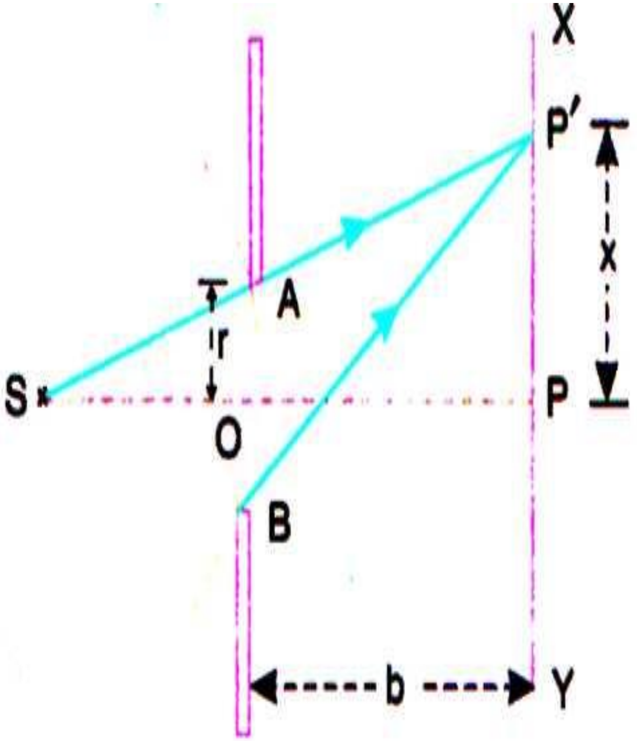
$$\delta = 2n\frac{\lambda}{2}$$

$$x_n = \frac{nb\lambda}{2r}$$

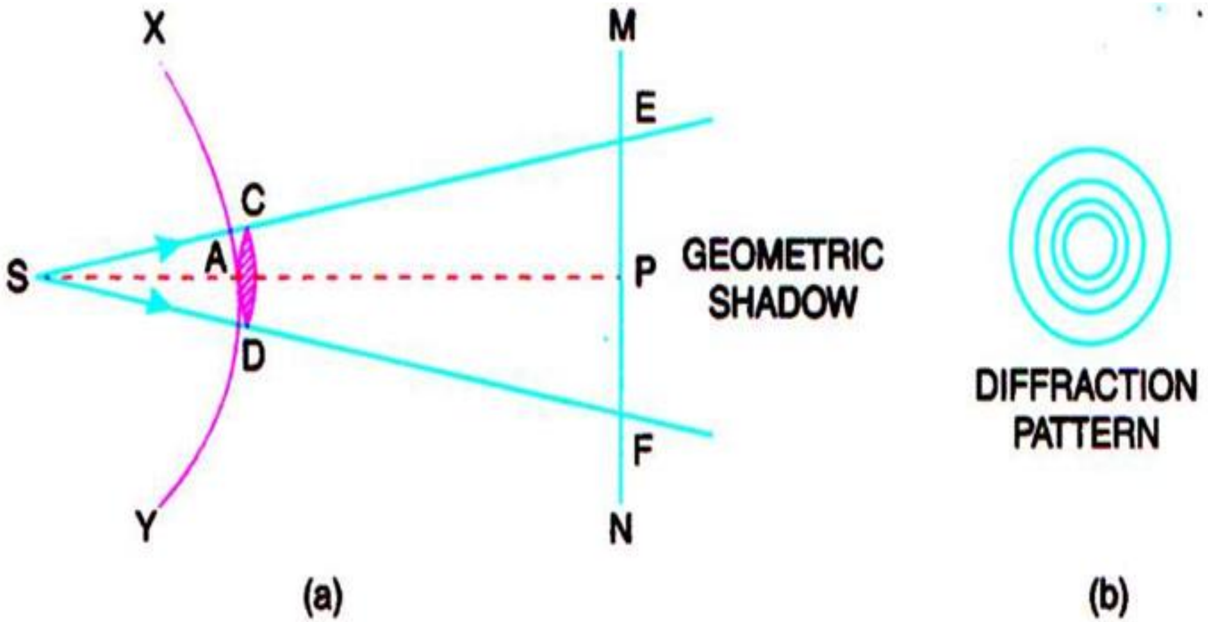
$$\delta = \frac{(2n+1)\lambda}{2}$$

$$\frac{(2n+1)\lambda}{2} = \frac{2rx_n}{b}$$

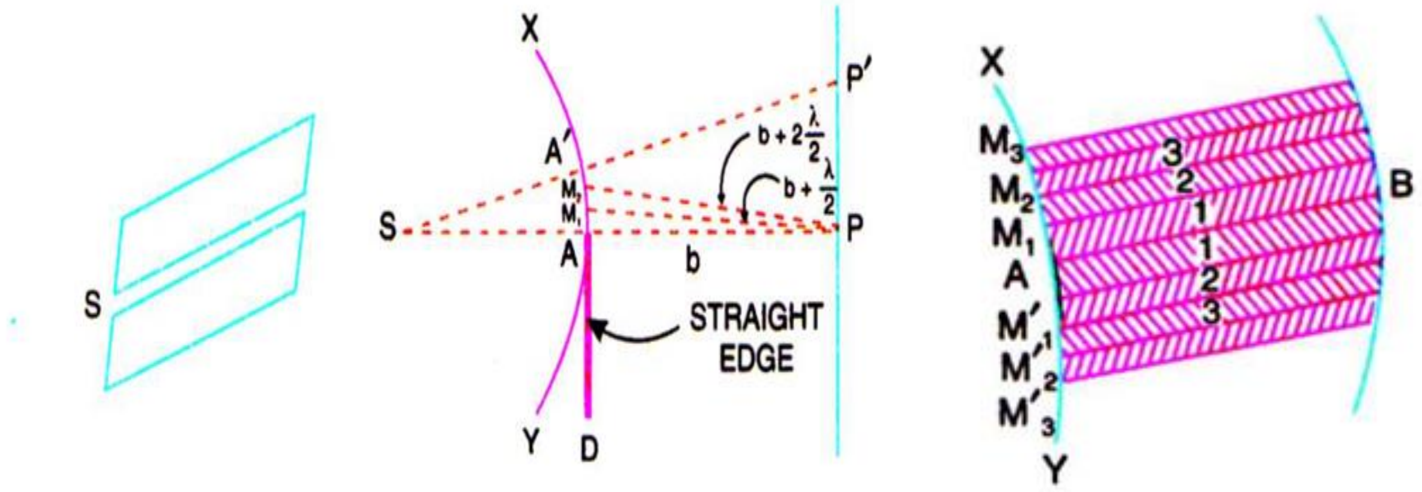
$$x_n = \frac{(2n+1)b\lambda}{4}$$

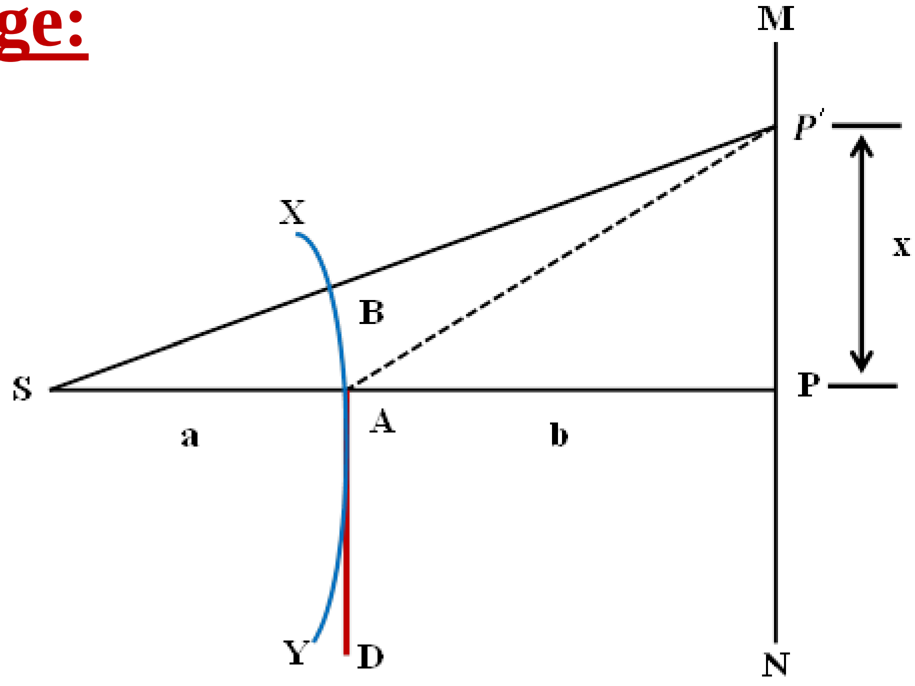


Diffraction at an Opaque Circular Disc:



Diffraction Pattern Due to a Straight Edge:





$$AP = b, \qquad PM_1 = b+\frac{\lambda}{2} \quad \text{and} \quad PM_2 = b+\frac{2\lambda}{2} \text{ etc.}$$

Positions of Maximum and Minimum Intensity:

$$\delta = AP' - BP'$$

$$\delta = \left(b^2 + x^2\right)^{1/2} - (SP' - SB)$$

$$\delta = \left(b^2 + x^2\right)^{1/2} - \left(\sqrt{(a+b)^2 + x^2} - a\right)$$

$$\delta = b\left(1 + \frac{x^2}{2b^2}\right) - (a+b)\left[1 + \frac{x^2}{2(a+b)^2}\right] + a$$

$$\delta = \left(b + \frac{x^2}{2b}\right) - \left[(a+b) + \frac{x^2}{2(a+b)}\right] + a$$

$$\delta = \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{(a+b)}\right) = \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)}\right)$$

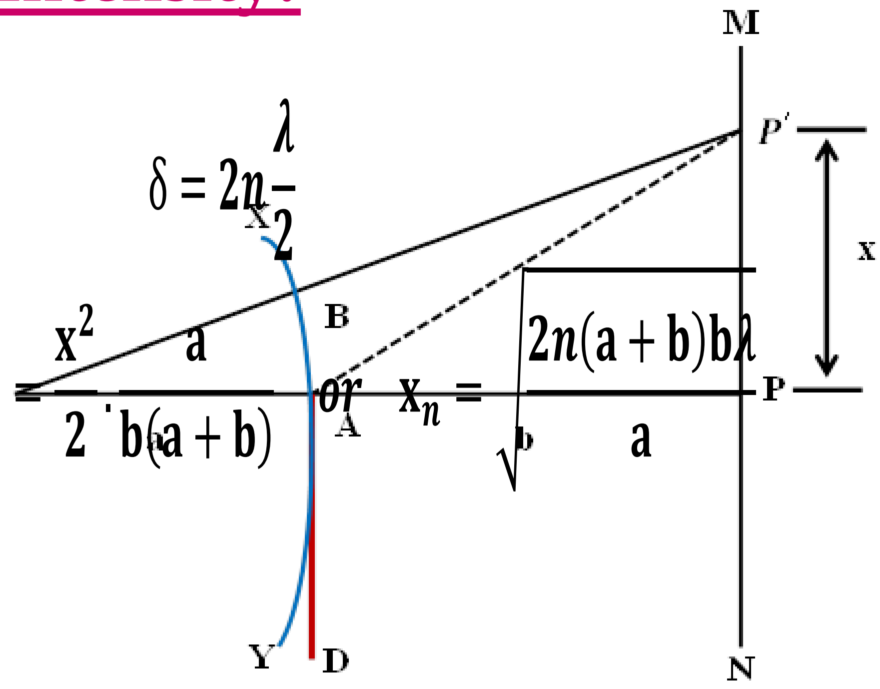
$$\delta = \frac{x^2}{2} \cdot \frac{a}{b(a+b)}$$

$$\delta = \frac{(2n+1)\lambda}{2}$$

$$\frac{(2n+1)\lambda}{2} = \frac{x^2}{2} \cdot \frac{a}{b(a+b)} \qquad x_n^2 = \frac{(2n+1)(a+b)b\lambda}{a} \qquad x_n = \sqrt{\frac{(2n+1)(a+b)b\lambda}{a}}$$

$$\delta = 2n\frac{\lambda}{2}$$

$$2n\frac{\lambda}{2} = \frac{x^2}{2} \cdot \frac{a}{b(a+b)} \quad or \quad x_n = \sqrt{\frac{2n(a+b)b\lambda}{a}}$$



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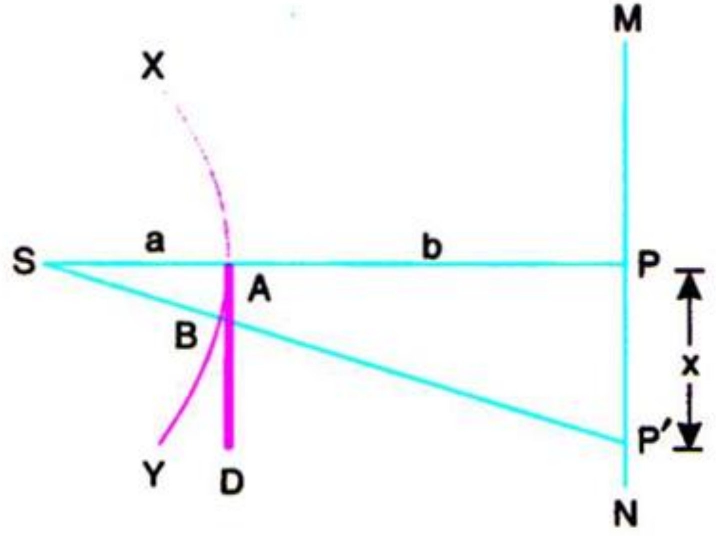
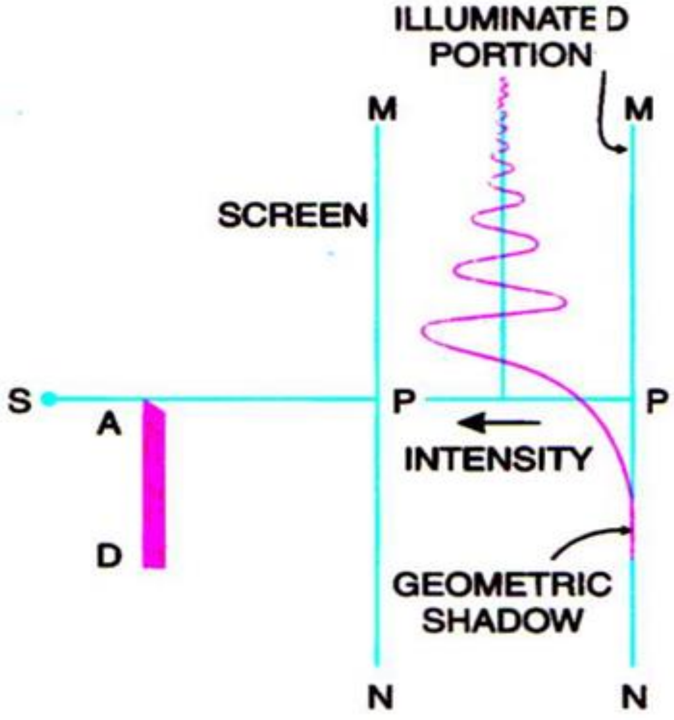
Intensity at a Point Inside the Geometrical Shadow (Straight Edge):





Example 1: What is the radius of the first half period zone in a zone plate behaving like a convex lens of focal length 60 cm for light of wavelength 6000 ?

Solution: Given that $f = 60$ cm $= 0.6$ m, $n = 1$, and $\lambda = 6000$ Å $= 6 \times 10^{-7}$m

$$f = \frac{r_n^2}{n\lambda} \quad \therefore \; r_n^2 = fn\lambda$$

$\therefore$ $$r_1^2 = 0.6\ \text{m} \times 1 \times 6 \times 10^{-7}\ \text{m} = 36 \times 10^{-8}\ \text{m}^2$$

or $$r_1 = 6 \times 10^{-4}\ \text{m} = \mathbf{0.6\ mm}.$$

Example 2: A circular aperture of 1.2 mm diameter is illuminated by plane waves of monochromatic light. The diffracted light is received on a distant screen which is gradually moved towards the aperture. The centre of the circular patch of light first becomes dark when the screen is 30 cm from the aperture. Calculate the wavelength of light.

Solution: Given that diameter = 1.2 mm = 0.12 cm

$$\text{Radius} = r = 0.06 \text{ cm}$$

$$b = 30 \text{ cm}$$

Here

$$(b^2 + r^2) = (b + \lambda)^2$$

$$30^2 + (0.06)^2 = (30 + \lambda)^2$$

$$\lambda = \frac{(0.06)^2 \text{ cm}^2}{2 \times 30 \text{ cm}} \quad (\lambda^2 \text{ neglected})$$

$$= 0.00006 \text{ cm} = \mathbf{6000\ Å}$$


تحياتي لكم


Thank You