1. **Results on Riemann Integral**

(7.1) **Theorem**: Let are bounded functions with Riemann integral, then we have

1. If .
2. If .
3. If .

**Proof:** (1) since .

partition of .

Since is Riemann integral on ,

(7.2) **Theorem**: Let is bounded function with Riemann integral, then is Riemann integral on and .

**Proof:** since is Riemann integral on **.**

is neglected set.

Since

is neglected set.

is Riemann integral on **.**

(7.3) **Theorem**: Let are bounded functions with Riemann integral, then we have

1. is Riemann integral on **.**
2. is Riemann integral on **.**
3. If bounded, then is Riemann integral on **.**

**Proof:** (1) let

Since bounded .

Since is Riemann integral on **.**

partition of

Also,

1. If

is Riemann integral on .

1. If

Since is Riemann integral on .

is Riemann integral on .

Since

is Riemann integral on .

But is Riemann integral on .

**Mean Value Theorem for Integral.**

(7.4) **Theorem**: Let continuous, then there is .

(7.5) **Example**: Let a function defined as , then , but .

(7.6) **Theorem**: Let continuous and non-negative. If , then .