1. **Lebesgue Integral**

(13.1) **Definition**: Let is measurable set in . Lebesgue partition of is a finite family of measurable subsets of and satisfies:

1. is measurable set in .
2. .
3. .

(13.2) **Examples:**

1. Let and , then is Lebesgue partition of .
2. If , then is Lebesgue partition of .

(13.3) **Definition:** Let and are Lebesgue partitions of . We said that is a refinement of (, if .

(13.4) **Theorem:** Let and are Lebesgue partitions of , then is Lebesgue partitions of and .

(13.5) **Definition**: Let is a bounded set and measurable in , and let is a bounded function and let is Lebesgue partitions of . Put

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, ,

Now, define

(13.6) **Theorem**: Let is a bounded set and measurable in , and let is a bounded function and let is Lebesgue partitions of , then .

**Proof:** since is a bounded function

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(13.7) **Theorem**: Let is a bounded set and measurable in , and let is a bounded function and let are Lebesgue partitions of . If , then

and then, .

(13.8) **Corollary**: Let is a bounded set and measurable in , and let is a bounded function and let are Lebesgue partitions of . If , then

**proof:** let is Lebesgue partitions of

and

(13.9) **Definition:** Let is a bounded set and measurable in , and let is a bounded function. Define

is Lebesgue partitions of

is Lebesgue partitions of

we note that are bounded, so we define

(13.10) **Definition:** Let is a bounded set and measurable in , and let is a bounded function. We said that is Lebesgue integrable on , if

(13.11) **Example:** Let is a bounded set and measurable in , and let is a function defined as , then is Lebesgue integrable on and

**Solution:** let is Lebesgue partition of .

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is Lebesgue integrable and .

(13.12) **Theorem:** If is Riemann integrable, then is Lebesgue integrable and .

**Proof:** since is measurable set and every Riemann partition of is Lebesgue of and

, also

since

since , and is Riemann integrable .

(13.13) **Example:** Let a function defined as , let ,

Then is Lebesgue partition of and .

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but is Lebesgue integrable on .

(13.14) **Theorem:** Let is a bounded set and measurable in , and let a functions are bounded and Lebesgue integrable on . If , then .

(13.15) **Theorem:** Let is neglected set in , then a bounded function is Lebesgue integrable on and .

**Proof:** let is Lebesgue partition of

Since

is Lebesgue integrable on and