(1-1) Stability and Abundance

When we examine the characteristics of stable nuclei, we find that for A<40 the number of protons equals the number of neutrons (N=Z). But for A≥40, stable nuclei have N=1.7Z; i.e., the number of neutrons is greater than the number of protons, see figure (1-1). This can be understood from the fact that, in larger nuclei, the charge density, and therefore the destabilizing effect of Coulomb repulsion, is smaller when there is a neutron excess. Furthermore, a survey of the stable nuclei reveals that even-even nuclei are the ones most abundant in nature. This again lends support to the strong-pairing hypothesis, namely that pairing of nucleons leads to nuclear stability. The most stability of nuclei has a magic numbers (2, 8, 20, 28, 50, 82, 126 and 184) of protons or neutrons to making a closed shell.

The numbers of stable nuclei in nature are:

For even-even = 156, even-odd = 48, odd-even = 50 and odd-odd = 5 (^{1}H, ^{6}Li, ^{10}B and ^{14}N).

The most important stability parameters:

1. Magic numbers
2. Pairing of nucleons
3. Equality and percentage between protons & neutrons
4. Mass number (A)

From these parameter, we can predict the stability and which of radiation may be emitting from the nucleus.

Example: How many atoms of ^{10}B are there in 5 grams of boron?

Sol.: From tables; the atomic weight of elemental boron W(B) = 10.811gm/mol. The 5gm sample of boron equals m/W(B) moles of boron, and since each mole contains N_a atoms, the number of boron atoms is:

$$N(B)=\frac{mN_a}{W(B)}=\frac{5gm(0.6022x10^{24}\,atoms/mol)}{10.811gm/mol}=2.785x10^{23}\,atoms$$

From tables, the isotopic abundance of ^{10}B in elemental boron is found to be 19.9%. The number $N(^{10}B)$ of ^{10}B atoms in the sample is therefore,

$N(^{10}B) = (0.199)(2.785x10^{23}) = 5.542x10^{22}$ atoms

Or $\mathbf{N(B)\approx\frac{mN_a}{A}}$, where $\mathbf{W(B)\approx mass\,number\,(A)}$

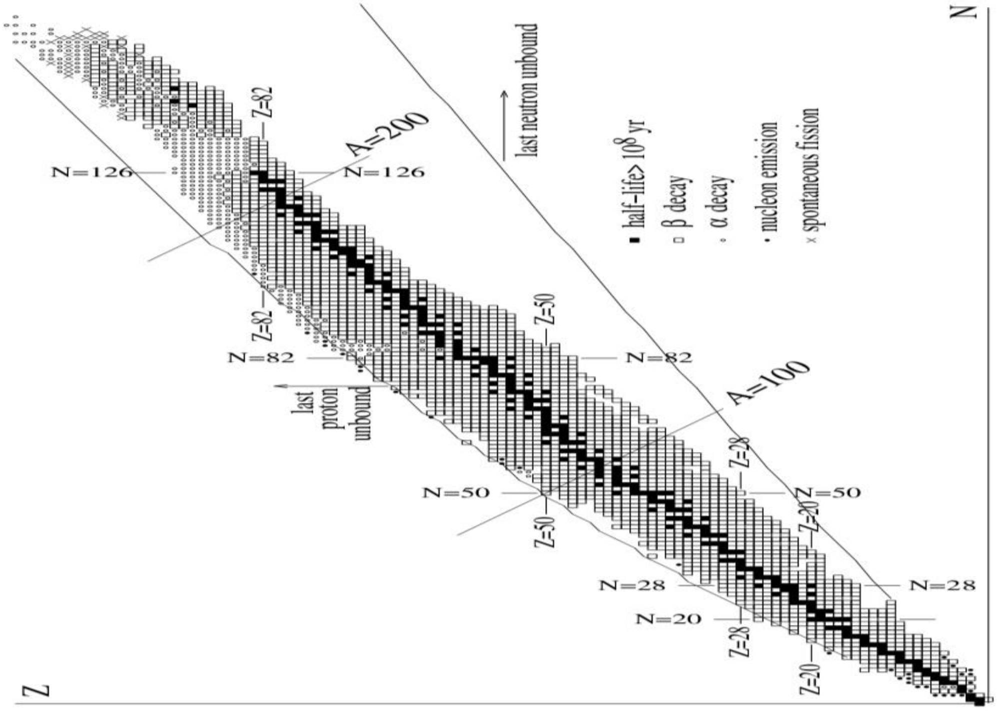

Figure (1-1): the nuclei, the black squares are long-lived nuclei present on earth. Unbound combinations of (N,Z) lie outside the line marked "last proton/neutron

(1-2) Nuclear Mass and Charge

Nuclear masses are measured in terms of the atomic mass unit : 1amu or 1u= $1.66x10^{-27}$kg. One u is equivalent to 1/12 of the mass of a neutral ground-state atom of ^{12}C. Since electrons are much lighter than protons and neutrons (protons and neutrons have approximately similar mass), one nucleon has mass of about 1amu. Because of the mass-energy equivalence, we will often express masses in terms of energy units. To convert between energy (in MeV) and mass (in u) the conversion factor is of course the speed of light square (since $E = mc^2$). In these units we have: $1u = 931.502 MeV/c^2$.

- Proton mass: $m_p = 1.007276u = 938.280 MeV/c^2$
- Neutron mass: $m_n = 1.008665u = 939.573 MeV/c^2$
- Electron mass: $m_e = 0.000549u = 0.511 MeV/c^2$

Mass difference between proton and neutron of order one part per thousand of u or, $(m_n - m_p)c^2 = 1.29$ MeV

For nuclear physics, the mass difference is much more important than the masses themselves. Also of great phenomenological importance is the fact that this mass difference is of the same order as the electron mass.

$m_e c^2 = 0.511$ MeV

We would expect the mass of the nucleus to be:

$M_N(A,Z) \approx Zm_p + Nm_n$

For atoms, $M_A(A,Z) \approx Zm_p + Zm_e + Nm_n$

However, the measured values of nuclear masses revel that the mass of a nucleus is smaller than the sum of the masses of its constituents. Namely,

$M_N(A,Z) = Zm_p + Nm_n - B$

Where B=B.E. is the nuclear binding energy=mass difference (ΔM)

This explains why an isolated nucleus cannot just fall apart into its constituents, because that would violate the principle of conservation of energy.

The mass difference comes from the energy gained in bringing the nucleons into their mutual potentials, is the mass defect (Δ) which written as:

$\Delta = [M(A,Z) - A]c^2$

mass excess = $[M(A,Z)-A]$

$$\text{packing fraction} = \frac{M(A,Z) - A}{A}$$

Which is negative, and can be thought of as being proportional to the nuclear binding energy (B.E.); the absolute value of Δ is related to the minimum energy required to break up the nucleus into its components.

Example: for ^{16}O (Z=8, N=8)

From tables, Atomic mass= 15.994915 u =15.994915 x 931.502 = 14899.295 MeV/c^2

$\Delta=[M(A,Z)-A]c^2$

$=(15.994915 - 16)c^2 =-0.005085\ c^2 x931.502MeV/c^2=-4.737MeV$

Atomic mass= $8m_p+8m_e+8m_n=15026.912\ MeV/c^2$

$\Delta M= [Zm_p + Zm_e + Nm_n - M(A,Z)]c^2$

ΔM=15026.912-14899.295=127.617 MeV

127.617/16=7.976 MeV per nucleon

Atomic nuclei are quantum bound states of particles called nucleons of which there are two types, the positively charged proton and the uncharged neutron. As far as we know, leptons are elementary particles that cannot be considered as bound states of constituent particles. Nucleons, on the other hand, are believed to be bound states of three spin 1/2 fermions called quarks. Two species of quarks, the up-quark u (charge 2/3) and the down quark d (charge -1/3) are needed to construct the nucleons:

proton = uud , neutron = udd .

q_p= -e = +1.6x10^{-19}C, nuclear charge $Q_N=Zq_p$

Besides protons and neutrons, there exist many other particles that are bound states of quarks and antiquarks. Such particles are called hadrons. For nuclear physics, the most important are the three pions: (π^+, π^0, π^-), that strong interactions between nucleons result from the exchange of pions and other hadrons just like the electromagnetic interactions which results from the exchange of photons.