(1-1) Nuclear Size and Density

The existence of the nucleus as the small central part of an atom was first proposed by Rutherford in 1911. Later, in 1920, the radii of a few heavy nuclei were measured by Chadwick and were found to be of the order of 10^{-14}m, much smaller than the order of 10^{-10}m for atomic radii.

$R = r_o A^{1/3} = 1.2A^{1/3}$ fm

$$r_o \approx \begin{cases} 1.4\text{fm} & \text{for nuclear particle scattering on nuclei} \\ 1.2\text{fm} & \text{for electron scattering on nuclei} \end{cases}$$

The density of nuclei is approximately constant, and those nucleons are tightly packed inside the nucleus.

$$\rho \propto \frac{A}{R^3} \quad , \quad R = r_o A^{1/3} \Rightarrow \rho \propto \frac{1}{(r_o)^3} = constant$$

The experiments have been performed and analyzed for a great many nuclei and at several incident electron energies. All the result can be approximately explained by a charge distribution given by:

$$\rho(r) = \frac{\rho_o}{1 + \exp[(r - R)/a]}$$

Where the physical significance of the various parameters are illustrated in figure (1-2).

$\rho_o \approx 1.65\text{x}10^{44}$ nucleons/m^3 =0.165 nucleons/fm^3

$R \approx 1.07A^{1/3}$ fm, $a \approx 0.55$fm, t is the surface region

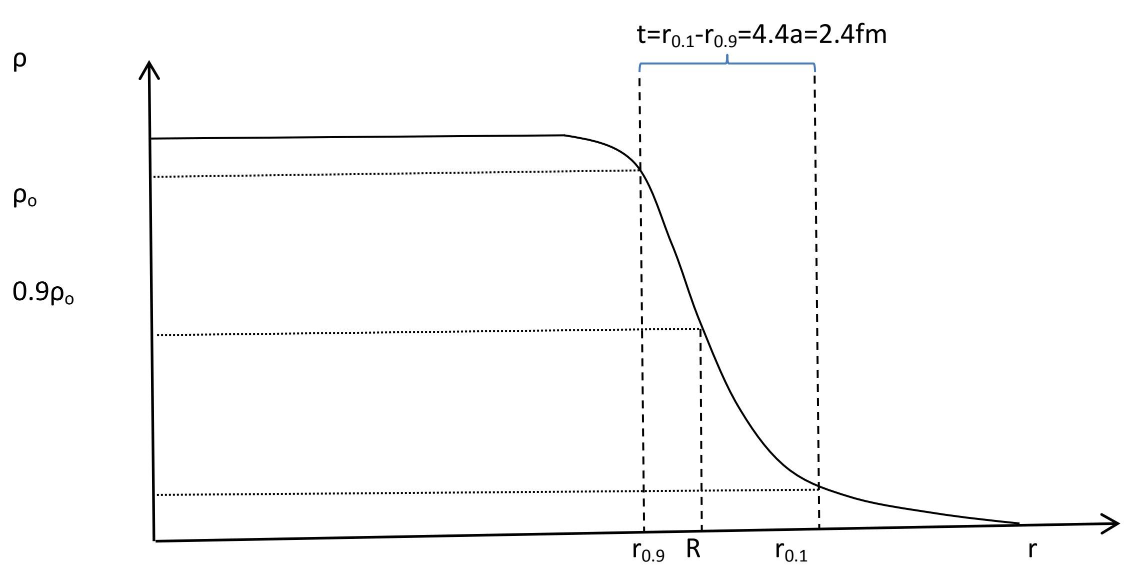


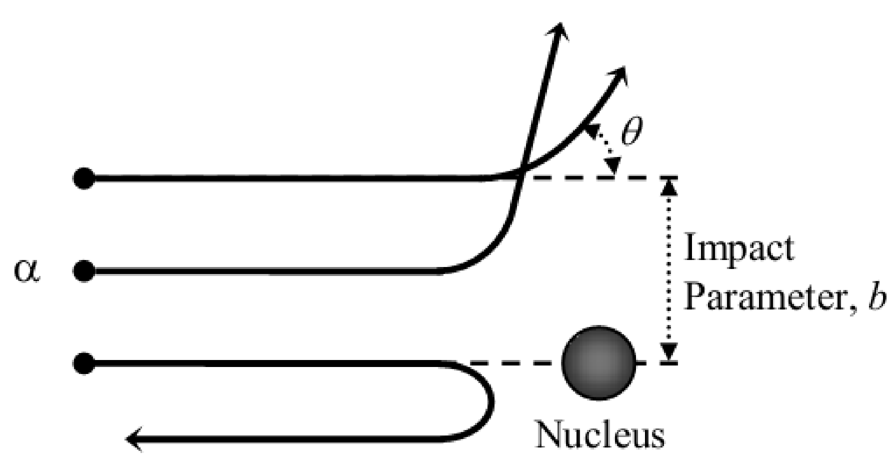
Figure (1-2): plot of nuclear density equation $\rho(r)$ Vs. r, the meaning of ρ_o, R and a are illustrated.

From the distribution of the α-particle's scattering angles, Rutherford concluded that the structure of an atom most likely mimics the solar planetary system. The size of the nucleus at the center of the atom was estimated based on the kinetic energy (T) of the incident α-particle and its potential energy at the point of closest approach (d). The closest approach occurs in the case of a head-on collision in which the α-particle comes to rest before it bounces back at an angle of 180 degrees, see figure (1-3). At that point the kinetic energy is zero, and the potential energy equals the initial kinetic energy.

$$\mathbf{T = \frac{k(Ze)(ze)}{d}}$$

$$\mathbf{b = \frac{k(Ze)(ze)}{T\tan(\theta/2)} = \frac{d}{\tan(\theta/2)}}$$

Where k is the Coulomb force constant = $1 / (4\pi\varepsilon_o) = 8.99\text{x}10^9\ Nm^2/C^2$.



Example: In Rutherford's experiment the kinetic energy of the incident α particles was 7.7MeV. Estimate the upper limit size of the gold nucleus and comment on the effect of increased energy of the incident particles in the experiment.

Sol.: the point of closest approach will determine the size of the nucleus. For the head-on collision it follows as:

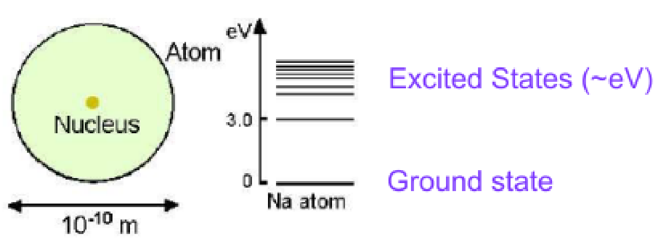
$$d = \frac{k(79e)(2e)}{7.7\,MeV} = \frac{(79)(2)ke^2}{7.7\,MeV} = 30\ fm$$

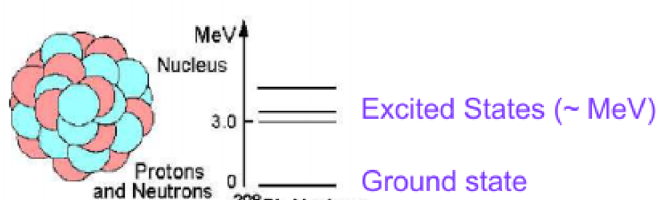
This implies that the gold nucleus has radius smaller than 30fm (the actual measurement is about 8fm).

If the incident energy of α particles in Rutherford's experiment is increased, some of the particles would penetrate the nucleus; first in the head-on collisions and then for smaller angles as the energy is further increased. The limiting kinetic energy for the incident particle above which the Rutherford experiment would not agree with theoretical explanation.

$$T \approx \frac{(79)(2)ke^2}{R} = 28.5\mathrm{MeV}$$

Where R represents the radius of the gold nucleus.





(1-2) Quantum theory of angular momentum

Orbital angular momentum (from relative motion) is quantized in units of ħ, where ħ=h/2π, h is Planck's constant, L = 0 ħ, 1 ħ, 2 ħ, ...

Spin (intrinsic angular momentum) denoted by s for single particle and I for nucleus (nuclear spin), can be either integral or half-integral:

Fermions have half-integral spin s= 1/2 ħ, 3/2 ħ, 5/2 ħ, ...

Typical fermions include electrons, protons, neutrons, quarks, neutrinos...

Bosons have integral spin s = 0 ħ, 1 ħ, 2 ħ, ...

Typical bosons include pions, photons, W- and Z-bosons, gluons, (gravitons).

Since angular momentum is a vector, the total angular momentum of a nucleus is the vector sum of the angular momentum of its constituents, we find experimentally that complex nuclei have intrinsic angular momentum equal to I ħ, where:

For even-A nuclei: I is an integer (including zero)

For odd-A nuclei: I is an integer (including zero) plus one-half

For even-even nuclei: I=0

For example, the nucleus of deuterium ^2H has I=1 and the nucleus of ^7Li has I=3/2.

In QM we can only discuss the total angular momentum J and one component, usually J_z (The other components are in determinate), J_z can take on the values

J_z = -J, -J+1, -J+2, ... J-1, J i.e. (-J→J), sometimes for J_z we write m. So the angular momentum for a particle (or system of particles) is denoted by (J,J_z) or (J,m_ℓ).

$$\vec{J} = \sum_i (\vec{\ell}_i + \vec{s}_i) = \sum_i \vec{j}_i$$

$$\vec{J} = \vec{L} + \vec{S}$$

where $\vec{L} = \sum_i \vec{\ell}_i$ and $\vec{S} = \sum_i \vec{s}_i$

Adding angular momentum: The Rules; Suppose we start with (J_1,m_1) and (J_2,m_2) and "add" them together. What is final (J,m_ℓ)?

(1) z-component is added: $m_\ell = m_1 + m_2$.

(2) $|J_1-J_2| \le J \le J_1+J_2$.

Parity:

Wave functions Ψ such that Ψ(-r) =Ψ (r) have even parity; wave functions such that Ψ (-r) = -Ψ (r) have odd parity. Parity is a quantum number and usually denoted by π. The parity of a single nucleon is: $\pi = (-1)^\ell$, the intrinsic parities of free nucleons are: $\pi_p = \pi_n = +1$

The Pauli Exclusion Principle: no two identical fermions can be in exactly the same quantum mechanical state.