Chapter Two

(Binding Energy)

(2-1) Nuclear Binding Energy

Since an atom contains Z positively charged particles (protons) and N=A-Z neutral particles (neutrons), the total charge of a nucleus is +Ze, where e represents the charge of one electron. Thus, the mass of a neutral atom, M_{atom}, can be expressed in terms of the mass of its nucleus, M_{nuc} and its electrons m_e.

$$M_{atom} = M_{nuc} + Zm_e \qquad M_{nuc} = Zm_p + (A-Z)m_n$$

where m_p is the proton mass, m_e the mass of an electron and m_n the mass of a neutron. For example the mass of the rubidium nucleus, ^{87}Rb, which contains 37 protons and 50 neutrons, can be calculated as:

$$M_{nuc}(^{87}Rb) = 37 \times 1.007277 + 50 \times 1.008665 = 87.7025 amu$$

The atomic mass, indicated on most tables of the elements, is the sum of the nuclear mass and the total mass of the electrons present in a neutral atom. In the case of ^{87}Rb, 37 electrons are present to balance the charge of the 37 protons. The atomic mass of ^{87}Rb is then:

$$M_{atom}(^{87}\mathrm{Rb}) = M_{nuc}(^{87}\mathrm{Rb}) + Zm_e$$
$$= 87.7025 + 37 \times 0.00055 = 87.7228 amu$$

From the periodic table, the measured mass of a ^{87}Rb atom is found to be $M_A^{measured}(^{87}Rb)$ = 86.909187 amu. These two masses are not equal and the difference is given by:

$$\Delta m = M_{atom}(^{87}Rb) - M_{atom}^{measured}(^{87}Rb) = 0.813613 amu$$

Expanding the terms in this equation, shows that the difference in mass corresponds to a difference in the mass of the nucleus

$$\Delta m = M_{atom} - M_{atom}^{measured}$$
$$= Zm_p + Zm_e + (A - Z)m_n - M_{nuc}^{measured} - Zm_e$$

which reduces to

$$\Delta m = M_{atom} - M_{atom}^{measured}$$
$$= Zm_p + (A - Z)m_n - M_{nuc}^{measured} = M_{nuc} - M_{nuc}^{measured}$$

Thus, when using atomic mass values given by the periodic table, the mass difference between the measured and calculated is given by

$$\Delta m = M_{nuc} - M_{nuc}^{measured} = Zm_p + Zm_e + (A - Z)m_n - M_{atom}^{measured}$$

Notice also that

$$Zm_p + Zm_e = Zm_H$$

Where m_H is a mass of the hydrogen atom.

From this and other examples it can be concluded that the actual mass of an atomic nucleus is always smaller than the sum of the rest masses of all its nucleons (protons and neutrons). This is because some of the mass of the nucleons is converted into the energy that is needed to form that nucleus and hold it together. This converted mass, Δm, is called the "mass defect" and the corresponding energy is called the "binding energy" and is related to the stability of the nucleus; the greater binding energy leads to the more stable the nucleus. This energy also represents the minimum energy required to separate a nucleus into protons and neutrons. The mass defect and binding energy can be directly related, as shown below:

$$\mathrm{B(A, Z)} = \Delta \mathrm{m} \times 931.5 \mathrm{MeV/amu} \qquad \text{or}$$
$$\mathrm{B(A, Z)} = 931.5(\mathrm{Zm_p} + \mathrm{Zm_e} + \mathrm{Nm_n} - \mathrm{M_{atom}^{measured}})$$

Since the total binding energy of the nucleus depends on the number of nucleons, a more useful measure of the cohesiveness is the average binding energy B_{ave}.

$$B_{ave}(A,Z) = \frac{B(A,Z)}{A} \quad (MeV / nucleon)$$

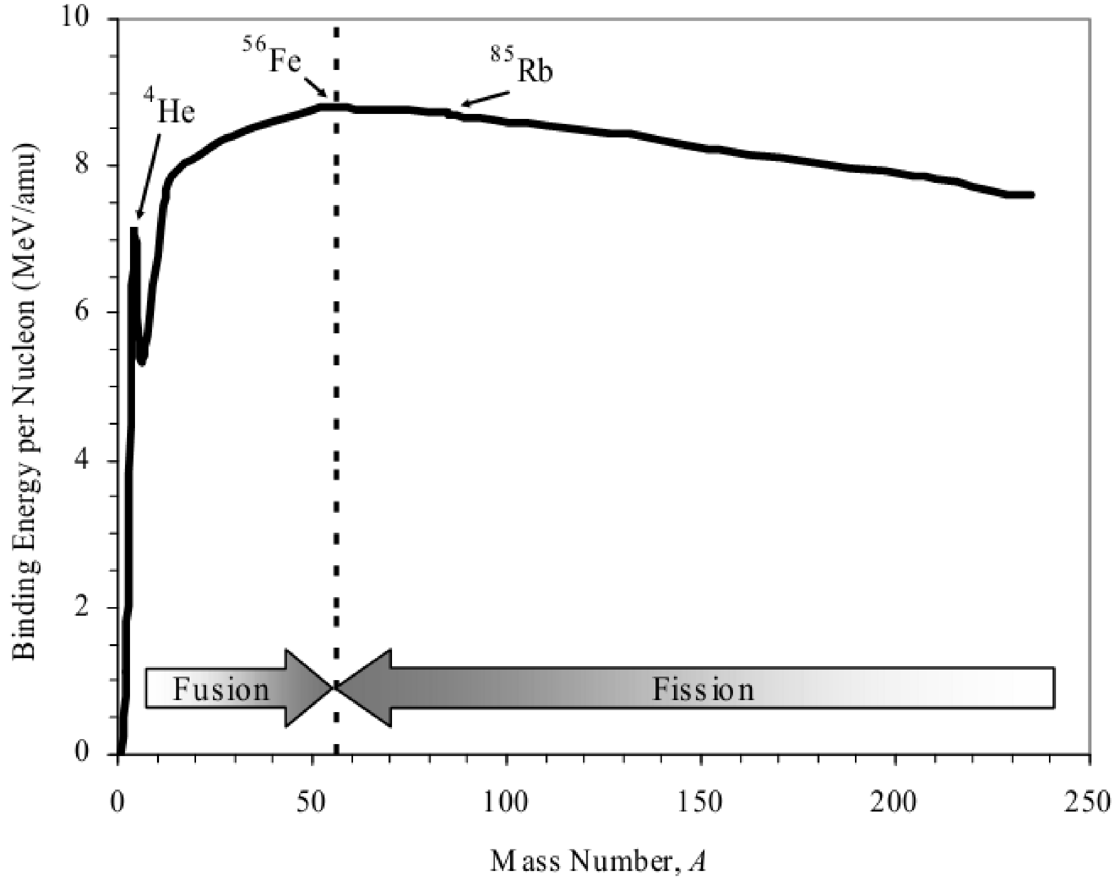

Figure (2-1): Variation of binding energy per nucleon with the atomic mass number

The binding energy per nucleon varies with the atomic mass number A, as shown in figure (2-1). For example, the binding energy per nucleon in a rubidium nucleus is 8.7MeV, while in helium it is 7.3MeV. The curve indicates three characteristic regions:

- Region of stability: A flat region between (A) equal to approximately 35 and 70 where the binding energy per nucleon is nearly constant. This region exhibits a peak near A = 60. These nuclei are near iron and are called the iron peak nuclei representing the most stable elements.

- Region of fission reactions: From the curve it can be seen that the heaviest nuclei are less stable than the nuclei near A = 60, which suggests that energy can be released if heavy nuclei split apart into smaller nuclei having masses nearer the iron peak. This process is called fission (the basic nuclear reaction used in atomic bombs as uncontrolled reactions and in nuclear power and research reactors as controlled chain reactions). Each fission event generates nuclei commonly referred to as fission fragments with mass numbers ranging from 80 to 160.
- Region of fusion reactions: The curve of binding energy suggests a second way in which energy could be released in nuclear reactions. The lightest elements (like hydrogen and helium) have nuclei that are less stable than heavier elements up to the iron peak. If two light nuclei can form a heavier nucleus a significant energy could be released. This process is called fusion, and represents the basic nuclear reaction in hydrogen (thermonuclear) weapons.