**2. Irrational and Real numbers**

Let be a complement set of in real number . , is called the set of irrational numbers, since .

(2.1)**Theorem:** Let and , then

1. .
2. , with .

**Proof:**(1) Assume that , since , since and is a field , also , but this is a contradiction.

2)Let , since is a field and , , also , but this is a contradiction.

(2.2)**Theorem:**(Density of irrational numbers)

Let an infinity irrational numbers between any two real numbers.

**Proof:** Since , since and are real numbers by using density of rational numbers since and . Now, since , by continuing this operation, we get on an infinite number of irrational numbers located between.

(2.3) **Definition**: Let , then

.

(2.4) **Note:** According to density of rational and irrational numbers, we can say that every interval of real numbers contains an infinite number of rational and irrational.

(2.5) **Definition**: (**Absolute Value**) Let be a real number, absolute value of is denoted by and defined as:

(2.6)**Theorem:**(Properties of Absolute Value)

1. max .
2. .
3. iff .
4. .
5. .
6. .
7. .
8. .
9. .
10. .
11. iff .

**Some Important Inequalities**

(2.7)**Theorem**:

1. **Cauchy-schwars Inequality.**

If . In particular, if and .

1. **Minkokowsks Inequality.**

If .

**Countable Sets.**

(2.8) **Definition**: Let be a sets. We say that is an equivalent to and written by , if there is a bijective function from into and written, if is an inequivalent to .

(2.9)**Theorem**:

1. If , then .

( defined by )

1. If , then .

( defined by )

1. If , then .

( defined by )

1. . ( defined by )
2. .

We deduce that are equivalent.

1. If , then .

( defined by )

1. If and , then .

( defined by )

1. If then .

( defined by )

1. If then.

( defined by )

1. If then .

( defined by )

1. If then .
2. For all put then
3. .
4. iff .
5. set.

(2.10) **Definition**: Let be a set. We say that is a finite set, if is a non-empty set or equivalent to for some . We say that is an infinite set, if does not finite set.

(2.11) **Definition**: If is a finite set, then for some and then there is a bijective function , put and then .

(2.12)**Theorem**:

1. Let be a non-empty sets such that then
2. is a finite iff is a finite.
3. is an infinite iff is an infinite.
4. For all finite set inequivalent to proper subset.
5. Every subset of finite set be a finite.
6. If is an infinite set and then is an infinite set.
7. If is an infinite set and is a set then is an infinite.

(2.13) **Definition**: Let is a set. We say that be a countable set, if be a finite or equivalent to . We say that be an infinite and countable, if be an infinite and equivalent to . We say that be an uncountable, if be an infinite and inequivalent to .

(2.14)**Theorem**:

1. Every finite set is a countable.
2. Each of be an infinite and countable set.
3. Each of and an intervals of are an uncountable sets.

(2.15) **Note**: If be an infinite countable set, then and then there is a bijective function , put and then .

(2.16)**Theorem**:

1. Every countable infinite set be an equivalent to a proper subset.
2. Every infinite set contains a countable infinite subset.
3. The set be a countable.
4. If are a countable sets, then
5. be a countable.
6. be a countable.