**4. Infinite Series**

Let a real sequence and . A sequence of partial sums is called an infinite series and its denoted by . We say that be a terms of infinite series and called of numbers be a partial sums of infinite series .

(4.1) **Definition**: Let be a real sequence and , we called of is an infinite series and denoted by .

(4.2) **Definition**: We say that is a convergent, if is a converge to , this means (), is called infinite series sum , this means . If is a divergent (i.e. does not exist).

(4.3) **Example**: Does convergent

and then is a convergent.

(4.4) **Theorem**: (some special infinite series)

1. is called geometric series and is a basis of series. is a convergent, if and otherwise its be a divergent.
2. is called a harmonic series and it’s a divergent.

**Proof:** (1) if does not convergent, if it’s a convergent, so it’s a bounded, this means , but this a contradiction (Archimedes property) is a divergent.

(2) , i.e. if does not Cauchy sequence does not convergent is a divergent.

(4.5) **Examples**:

1. is a convergent, since and .
2. is a divergent, since .
3. is a convergent, since and .
4. is a convergent, since .
5. The number is a convergent, let .

(4.6) **Theorem**: Let and be a convergent infinite series, then

1. is a convergent and .
2. is a convergent and .

**Proof:** (1) Let and , since and be a convergent infinite series be a convergent , .

(4.7) **Corollary**: If is a convergent and is a divergent, then

1. is a divergent.
2. is a divergent .

**Proof:** (1) Suppose that is a convergent, since is a convergent is a convergent.

Since is a convergent, but this is a contradiction.

(4.8) **Example**: and are a divergent, but is a convergent.