**5. A Converging Test**

(5.1) **Theorem**: If be a convergent, then .

**Proof:** let , since is a convergent is a convergent and then is Cauchy sequence , if , , , .

(5.2) **Note**: If not necessary be a convergent, for example is a divergent, while .

(5.3) **Corollary:**

1. If is a divergent.
2. If is a convergent and is a divergent.

(5.4) **Example**: Show that be a divergent.

is a divergent.

(5.5) **Definition**: We say that be a dominate on , if .

(5.6) **Theorem:** Let be a non-negative of terms and is a dominate on or

1. If is a convergent is a convergent.
2. If is a divergent is a divergent.

**Proof;** (1) let , since is a convergent is a convergent and is a bounded , since is a bounded, since does not increasing and then is a convergent is a convergent.

(2) let is a convergent is a convergent, but this is a contradiction is a divergent.

(5.7) **Example:** Discuss a convergent of 1. 2. .

(1) since , since is a convergent is a convergent.

(2) since , since is a divergent is a divergent.

(5.8) **Example**: Let , if is a convergent, then .

, since is a convergent .

(5.9) **Definition**: Let , we said a series.

(5.10) **Theorem:** is a convergent, if and it’s a divergent, if .

**Proof;** (1) if is a divergent.

(2) if , since is a divergent is a divergent is a divergent where .

(3) if , we compare with geometry series such , since is a convergent, since is a convergent.

(5.11) **Theorem:** Let , if and be both a convergent or a divergent.

(5.12)**Corollary:** Let , if and is a convergent, then is a convergent.

**Ratio Test**

(5.13) **Theorem:** Let and converges to (i.e. ), then be

1. A convergent, if .
2. A divergent, if .
3. Where , then the test will be failed.

**Proof:** (1) If (by density of rational number) , since , let , since is a convergent is a convergent. (by ratio test).

(5.14) **Theorem:** Let , if is a convergent.

(5.15) **Example:** Discuss a convergent of (1) (2) (3) .

(1) and

is a convergent.

(2) and

is a divergent.

(3) and

is a convergent.

**The Root Test**

(5.16) **Theorem:** Let .

1. If is a convergent.
2. If is a divergent.

(1)Let , since is a convergent is a convergent (by root test).

(5.17) **Example:** Discuss a convergent of (1) (2)

(1) and is a convergent is a convergent.

**Integral Test**

(5.18) **Theorem:** Let be a continuous function which positive, decreasing and defined let is a convergent iff is a convergent.

(5.19) **Example:** Does convergent?

**Solution:** isa continuous and positive. is a decreasing and is a positive.

.

is a convergent.

**Alternations Series**

(5.20) **Definition**:We said that is an alternation, if .

(5.21) **Theorem:** If and is a convergent.

(5.22) **Definition**:We said that be an absolutely convergent, if is a convergent.

(5.23) **Definition**:We said that be a conditionally convergent, if is a convergent but is a divergent.

(5.24) **Examples:**

1. is an absolutely convergent, since is a convergent.
2. is a convergent, but is a divergent is a conditionally convergent.