**6. Metric Spaces**

(6.1) **Definition**: If be a non-empty set. We said that is a metric function on , if

1. .
2. .
3. .
4. .

(6.2) **Note**: is called Metric Space.

(6.3) **Example**: Let is a function defined by , then is a metric function, and is an usual metric space.

**Solution:** (1) let .

(2) .

(3) let .

(4) let

is a metric function on .

(6.4) **Example**: Let is a function defined by , does a metric function on?

**Solution:** let this means the second axiom does not satisfy does not metric.

(6.5) **Example**: Let be a non-empty set and is a function defined by for all , then is a metric function and is called discrete metric space.

**Solution:**

(1) since or .

(2) let , if (by the definition of a function ), if , since if , but this is a contradiction .

(3) let .

(4) let

a. if , since .

b. if , so or , let since is a metric function on .

(6.6) **Example**: Let be a non-empty set and is a function defined by for all , then is a metric function and is called indiscrete metric space.

(6.7)**Theorem:** Let be metric space, then

1. .
2. .

**Proof:**

(1)

Also

from

.

(6.8)**Theorem:** Let be a non-empty set, then a function be a metric function iff

1. iff .
2. .

**Proof:**  suppose that is a metric function are satisfy (from a definition).

if are satisfy,

(1)Let from , we get

, but from .

(2) same the condition .

(3) Let from , we get

.

(4) Let

is a metric function on .

(6.9)**Definition:** Let be metric space and . The diagonal of denoted by and defined by sup , if or contains on only one element, then . The distance of point from denoted by and defined by inf .

(6.10)**Note:** Its clear, if , then , and if , then .

(6.11)**Definition:** Let . The distance between is denoted by and defined by inf . Its clear that, if, then .

(6.12)**Example:** Let be usual metric space and , we note that .

(6.13)**Theorem:** Let be metric space and , then

1. .
2. If is a finite, then .
3. If , then .
4. .
5. If , then .

**Proof:**

(1) and (2) from the definition.

(3) since and

inf

, but

(6.14)**Notes:**

1. If , then not necessary that .
2. If , then not necessary that .