**12. Continuity**

(12.1)**Definition**: If be metric spaces. We said that a function is continuous at , if open set contains an open set contains .

(12.2)**Theorem:** Let be metric spaces, then a function be continuous at point open ball in an open ball in .

This means, .

(12.3)**Example:** Let be usual metric space. Prove that a function defined by is a continuous.

**Solution:** let .

Since

So,

Since

If

From , we get

Take min

Now, let and

From , we get

is a continuous at is a continuous.

(12.4)**Example:** Let be usual metric space. Prove that a function defined by is a continuous at .

**Solution:** let (since )

If

Choose min

Now, let and

is a continuous at .

(12.5)**Example:** Let be usual metric space. Prove that a function defined by is a continuous on .

(12.6)**Theorem:** Let be metric spaces, and a function, then the following properties are equivalent:

1. A function is a continuous.
2. If an open set , then be an open set in .
3. If a closed set , then be a closed set in .
4. .
5. .
6. .

(12.7)**Theorem:** Let be metric spaces, and be a continuous functions, then a function be a continuous function.

**Proof:** let is an open set in .

Since a function is a continuous is an open set in .

Since a function is a continuous is an open set in , but

is an open set in

be a continuous function.

(12.8)**Example:** Let be metric spaces, and a function. Prove that

1. If is a constant, then is a continuous.
2. If be discrete, then is a continuous.
3. If be indiscrete, then is a continuous.

**Solution:** (1) since is a constant, then .

Let be an open set in .

Since be an open sets be an open set in is a continuous.

**Sequentially Continuity**

(12.9)**Definition:** Let be metric spaces. We said a function be sequentially continuity at , if every sequence in in .

(12.10)**Theorem:** Let be metric spaces, then a function be a continuous at be sequentially continuity at .

(12.11)**Example:** Let be usual metric space. Prove that a function defined by is a discontinuous at .

**Solution:** take in and ,

since

is a discontinuous at