3. Solar Geometry

Modeling the performance of solar energy conversion systems requires the knowledge of the irradiance on a surface of given slope and orientation in different time scales (annual, monthly, daily or even hourly). To achieve this needs the evaluation of several geometrical factors. If the influence of the atmosphere is set aside for a moment, and second-order effects as reflection from adjacent surfaces and obstruction by neighboring structures are not considered, the irradiance is merely determined by

- the position of the sun in the sky and
- the slope and orientation of the surface.

For surfaces, which are not oriented perpendicular to the direction of the sun, the irradiance is given by (Fig. 3.1):

$$G = G_n \cos\theta, \tag{3.1}$$

where G_n is the irradiance on a plane normal to the sun's direction.

Generally, the angle of incidence θ has to be determined for a calculation of the irradiance of an arbitrary oriented surface. Therefore, the geometry of the sun-earth system as well as the surface orientation has to be considered.

Trigonometric relationships describing the influence of the earth's revolution around the sun and the earth's rotation around its own axis ($\longrightarrow$ diurnal changes of irradiance)

Annual variations of the irradiance are mainly caused by the varying position of the polar axis with respect to the sun (in addition to the varying sun-earth distance).

The earth revolves around the sun in a plane called *ecliptic plane*. The earth's axis (polar axis) is inclined at 23.45° (constant in time) with respect to the normal to the ecliptic plane. The same then holds for the angle between the earth's equatorial plane and the ecliptic plane.

The varying solar influence can be described by the angle between the earth's equatorial plane and the plane of its revolution around the sun. This angle is called

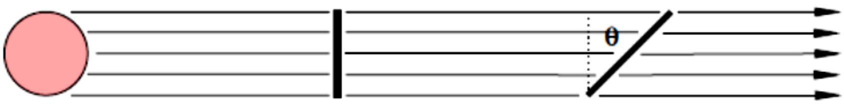

Fig. 3.1. Angle of incidence.

the ***solar declination*** δ. Its maximum daily change is less than 0.5° (occuring at the equinoxes), so that for practical purposes a constant value for a given day can be used:

$$\delta \simeq 23.45^\circ \cdot \sin\left(\frac{360(284+n)}{365}\right). \tag{3.2}$$

The following equation gives the declination, in degrees, with an accuracy of 0.05° (Spencer, 1971):

$$\begin{aligned}\delta = {} & (0.006918 - 0.399912\cos d + 0.070257\sin d \\ & -0.006758\cos 2d + 0.000907\sin 2d)\cdot 180/\pi,\end{aligned} \tag{3.3}$$

where $d = 2\pi(n-1)/365$ is the day angle in radians.

3.1 Solar Time

Daily variations of solar radiation are usually calculated on the basis of solar time, which is defined in the following way:

A ***solar day*** is the time interval between two consecutive crossings of the sun's path with the local meridian. The length of this interval changes from day to day (deviation < 30 sec). Only its mean value equals 24 h.

Solar noon then is the time of the crossing of the sun's path with the local meridian.

The variation of the solar day length is caused by (i) the elliptical path of the earth around the sun (Kepler's law: Earth sweeps equal areas in equal times) and (ii) the tilt of the earth's axis with respect to the ecliptic plane. This difference between

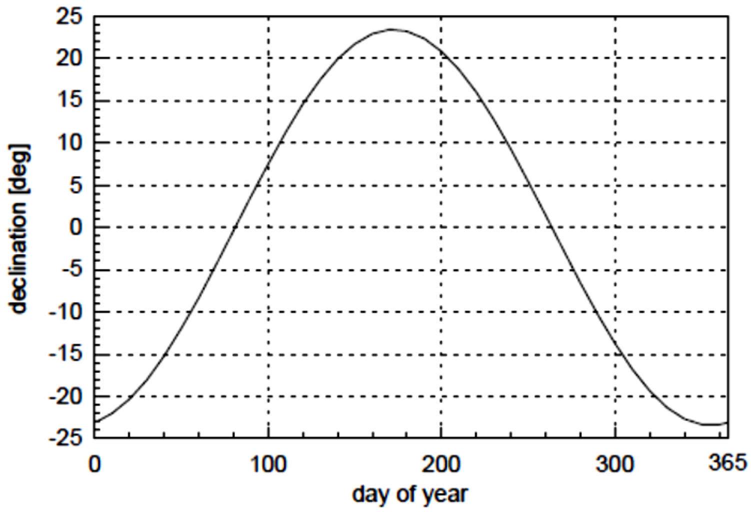

Fig. 3.2. Annual variation of the solar declination.

the solar time and the local mean time in minutes is expressed by the empirical *equation of time* E (Fig. 3.3):

$$E \simeq 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B, \tag{3.4}$$

where $B = 360 \cdot (n - 81)/364$ in degrees.
For a higher accuracy the following formula can be used (Spencer, 1971):

$$\begin{aligned} E = &(0.000075 + 0.001868 \cos d - 0.032077 \sin d \\ &-0.014615 \cos 2d - 0.040849 \sin 2d) \cdot (180 \cdot 4/\pi) \end{aligned} \tag{3.5}$$

where d again is the day angle.

The maximum value of E is 16.5 minutes (for day n = 303).

Solar time differs from standard time (i.e. the time we are used to work with, determined by the time zone) due to (i) variations of the length of the solar day and (ii) a difference between the local longitude and the standard longitude of the appropriate time zone (Fig. 3.4).

$\longrightarrow$ solar time = local time + E

Note: local time $\neq$ standard time !

The local time is a function of the actual (local) longitude L_l (i.e.: same local time only on the same meridian), the standard time is a function of timezones only (corresponding to standard longitudes L_s). Usually, the standard meridians are multiples of 15° E or W of Greenwich. The standard meridian for Central Europe, for example, is 15° E (TZ = -1 from Greenwich).
The true solar time (TST or LAT) is calculated from local time (LST) using

$$\begin{aligned} LAT &= LMT + E \\ &= LST - DST + 4(L_s - L_l) + E \end{aligned} \tag{3.6}$$

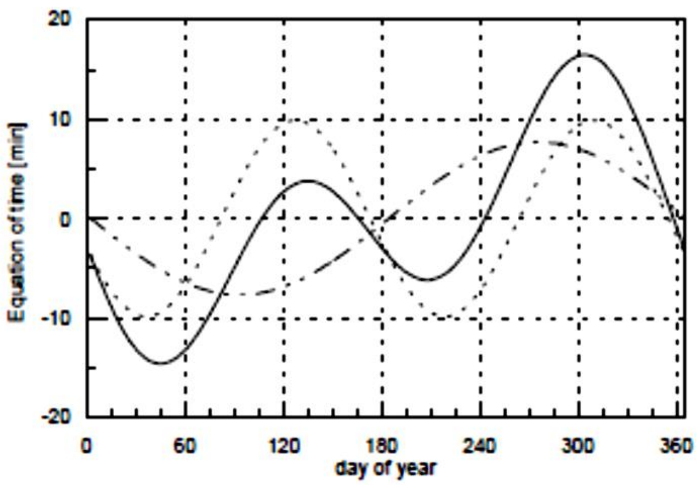

Fig. 3.3. The equation of time.

where DST = 1 hour during daylight saving time and = 0 otherwise.

Example: Calculation of the local time at solar noon for Oldenburg, October 15. The local longitude is 8.2° East (= - 8.2°), the standard longitude is 15° East (CET).

$$\begin{aligned} LST &= LAT + DST - 4(L_s - L_l) - E \\ &= 12:00 + 1 - 4(-15 - (-8.2)) - 14 \\ &= 13:00 + 0:27 - 0:14 \\ &= 13:13. \end{aligned}$$

The *hour angle* ω is a quantity which describes the solar time in trigonometric relationships. It equals the angular displacement of the sun from the local meridian due to the rotation of the earth. One hour corresponds to an angle of 15° (360°/24h). The morning hours are negative and the afternoon hours are positive by convention. At solar noon ω equals 0°.