3.2 Position of the Sun

To calculate the irradiance on any plane the position of the sun with respect to that plane (precisely: to the normal to that plane) must be known. The sun's position in the sky hemisphere can be completely described by two quantities (Fig. 3.5):

- solar altitude α (elevation above horizon)
- solar azimuth ψ

The sun's altitude is given by spherical trigonometry (with geographical latitude ϕ):

$$\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \tag{3.7}$$

The solar azimuth is given by:

$$\cos\psi = \frac{\sin\alpha\sin\phi - \sin\delta}{\cos\alpha\cos\phi} \tag{3.8}$$

The quantity describing the angle between the incoming solar beam radiation and the normal to the receiving surface is the angle of incidence θ (Fig. 3.5).
This angle depends on

- geographical location (latitude)

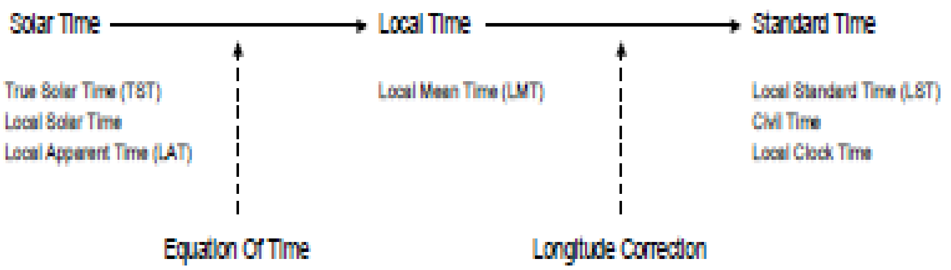

Fig. 3.4. Commonly used terms of time and their relationship.

– time of year (declination), time of day (hour angle)
– orientation of the plane (slope β, surface azimuth γ):

$$\theta = f(\phi, \delta, \omega, \beta, \gamma). \tag{3.9}$$

For its calculation, two additional angles have to be introduced: The slope angle β between the collector plane and the horizontal surface which varies between $0°$ for a horizontal plane and $90°$ for a vertical plane. The surface azimuth angle γ as the deviation of the normal of the plane from the local meridian. γ is counted clockwise from N where its value is $0°$ (thus for S it is $180°$) on both hemispheres. Note that in the literature often a value of $180°$ for an orientation towards the equator is used! Then it is:

$$\begin{aligned}\cos\theta = {} & \sin\delta\,(\sin\phi\cos\beta + \cos\phi\sin\beta\cos\gamma) \\ & + \cos\delta\cos\omega\,(\cos\phi\cos\beta - \sin\phi\sin\beta\cos\gamma) \\ & - \cos\delta\sin\beta\sin\gamma\sin\omega)\end{aligned} \tag{3.10}$$

In case of some special situations simplified expressions can be given:

- Horizontal surfaces: $\beta = 0°(i.e., \cos\beta = 1, \sin\beta = 0)$

$$\cos\theta = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \tag{3.11}$$

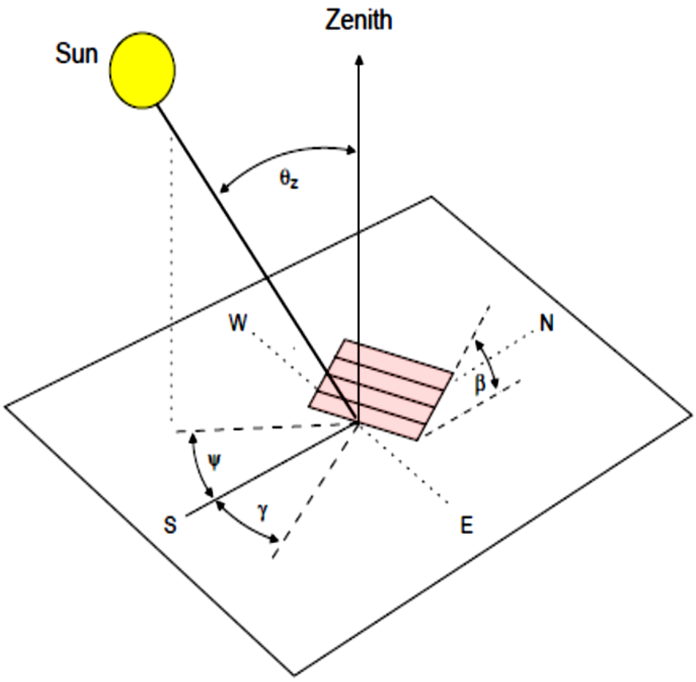

Fig. 3.5. Zenith angle θ_z, slope β, surface azimuth angle γ and solar azimuth angle ψ for a tilted surface.

For horizontal surfaces θ equals the zenith angle θ_z (see Fig.3.5).
The $90°$ complement of the zenith angle is the solar elevation angle α (solar altitude).

- Vertical surfaces facing towards the equator: $\beta = 90°, \gamma = 180°$ for the northern hemisphere and $\gamma = 0°$ for the southern hemisphere

$$\cos\theta = -\sin\delta\cos\phi + \cos\delta\sin\phi\cos\omega \tag{3.12}$$

- Inclined surfaces facing towards the equator with a tilt angle equal to the absolute value of the latitude (northern hemisphere: $\beta = \phi, \gamma = 180°$; southern hemisphere, $\beta = -\phi, \gamma = 0°$):

$$\cos\theta = \cos\delta\cos\omega \tag{3.13}$$

For solar noon ($\omega = 0°$) it is: $\theta = |\delta|$.

- Sunrise and sunset: $\theta_z = 90°$

$$\sin\delta\sin\phi = -\cos\delta\cos\phi\cos\omega$$

In this case, the sunset hour angle ω_{ss} is:

$$\cos\omega_{ss} = -\frac{\sin\delta\sin\phi}{\cos\delta\cos\phi} = -\tan\delta\tan\phi$$

Example: Daylength for winter solstice (21 December) in Oldenburg, Germany:
$\phi = +53.2°, \delta = -23.45°$
$\cos\omega_{ss} = -\tan(53.2°)\tan(-23.45°) = +0.58$
$\omega_{ss} = -54.56°$
The angle between sunrise and sunset then is $2|\omega_{ss}| = 109.12°$ resulting in a theoretical sunshine duration of $109.12°/15° = 7.27$ hours for that particular day.

3.3 Example: Extraterrestrial Radiation on a Horizontal Surface

- Instantaneous value:

$$\begin{aligned} G_o &= G_{on}\cos\theta_z \\ &\simeq G_{sc}\left\{1 + 0.033\cdot\cos\left(\frac{360\cdot n}{365}\right)\right\}\cos\theta_z \end{aligned} \tag{3.14}$$

- For a time period (t_1,t_2) with corresponding hour angles ω_1 and ω_2 Eq. 3.11 and integration gives:

$$\begin{aligned} I_o &= \int_{t_1}^{t_2} G_o dt = \int_{\omega_1}^{\omega_2} G_o \frac{12\cdot 3600}{\pi} d\omega \\ &= \frac{12\cdot 3600}{\pi} G_{sc}\left\{1 + 0.033\cos\left(\frac{360\cdot n}{365}\right)\right\}\cdot \\ &\quad \cdot\left(\cos\phi\cos\delta(\sin\omega_2 - \sin\omega_1) + \frac{\pi(\omega_2-\omega_1)}{180}\sin\phi\sin\delta\right), \end{aligned}$$

-

where $dt = (24\mathrm{h}/2\pi)d\omega$. I_o is in Jm^{-2}.

- Similarly, the daily extraterrestrial radiation is given in Jm^{-2}:

$$\begin{aligned} H_o &= \int_{-\omega_{ss}}^{\omega_{ss}} G_o dt \\ &= \frac{24 \cdot 3600}{\pi} G_{sc} \left\{ 1 + 0.033 \cos\left(\frac{360 \cdot n}{365}\right)\right\} \cdot \\ &\quad \cdot \left(\cos\phi \cos\delta \sin\omega_{ss} + \frac{\pi\omega_{ss}}{180} \sin\phi \sin\delta\right) \end{aligned}$$

where n is again the number of the day in the year and ω_{ss} is in degrees.