GOMETRIC OPTICS
LECTURE2

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2.1. PROPERTIES OF LIGHT

Reflection, refraction, dispersion and velocity are the important properties of light. We briefly discuss about them here.

2.1.1. REFLECTION OF LIGHT

When light travelling in a medium encounters a boundary leading to a second medium, part of the incident light is returned to the first medium from which it came. This phenomenon is called reflection. Reflection of light from a smooth surface is called regular or specular reflection see figure 1a. If the reflecting surface is rough, as shown in Figure 1b, the surface reflects the rays not as a parallel set but in various directions. Reflection from any rough surface is known as diffuse reflection. A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the incident light.
The difference between diffuse and specular reflection is a matter of surface roughness. In the study of optics, the term reflection is used to mean specular reflection.

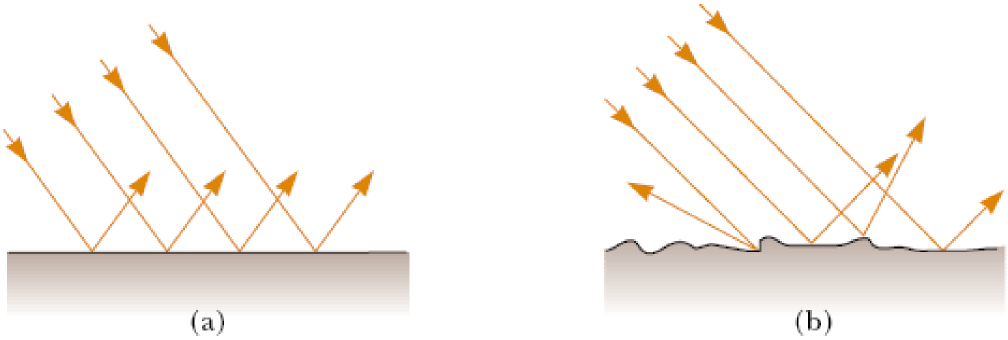


Fig 1 : Schematic representation of (a) specular reflection, where the reflected rays are all parallel to each other, and (b) diffuse reflection, where the reflected rays travel in random directions.

2.1.1.1. Laws of reflection

First law: The incident ray, the reflected ray and the normal at the of point incidence are in the same plane. This plane is called plane of incidence.
Second law: The angle of reflection equals the angle of incidence.

$\theta_1 = \theta'_1$ ……………………………………………………………(1)

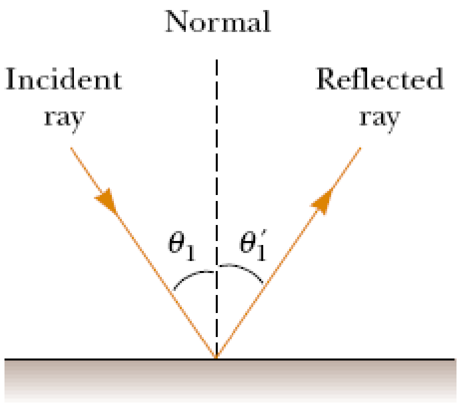


Fig 2. : According to the law of reflection, $\theta_1 = \theta'_1$. The incident ray, the reflected ray, and the normal all lie in the same plane.

2.1.2. REFRACTION OF LIGHT

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, as shown in **Figure 3**, part of the energy is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be **refracted**.

2.1.2.1. LAWS OF REFRACTION

First law: The incident ray, the refracted ray and the normal at the of point incidence lie in the same plane. This plane is called plane of incidence.

Second law: The ratio of the sine of the angle of incidence to the sine of the angle of refraction for any two given media is constant.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \text{constant} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Where v_1 is the speed of light in the first medium and v_2 is the speed of light in the second medium.

The **angle of refraction** in **Figure 3**, depends on the properties of the two media and on the angle of incidence.

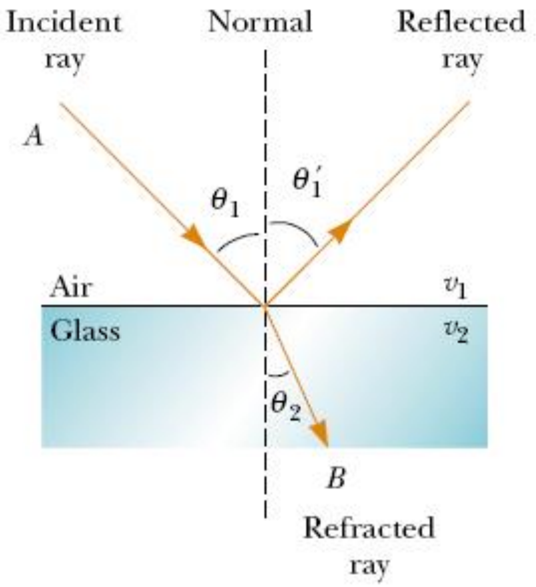


Fig 3.: A ray obliquely incident on an air–glass interface. The refracted ray is bent toward the normal because $v_2 < v_1$. All rays and the normal lie in the same plane.

From Equation 2, we can infer that when light moves from a material in which its speed is high to a material in which its speed is lower, as shown in Figure 1.8.a, the angle of refraction θ_2 is less than the angle of incidence θ_1, and the ray is bent ***toward*** the normal. If the ray moves from a material in which light moves slowly to a material in which it moves more rapidly, as illustrated in Figure 4.b, θ_2 is greater than θ_1, and the ray is bent ***away*** from the normal.

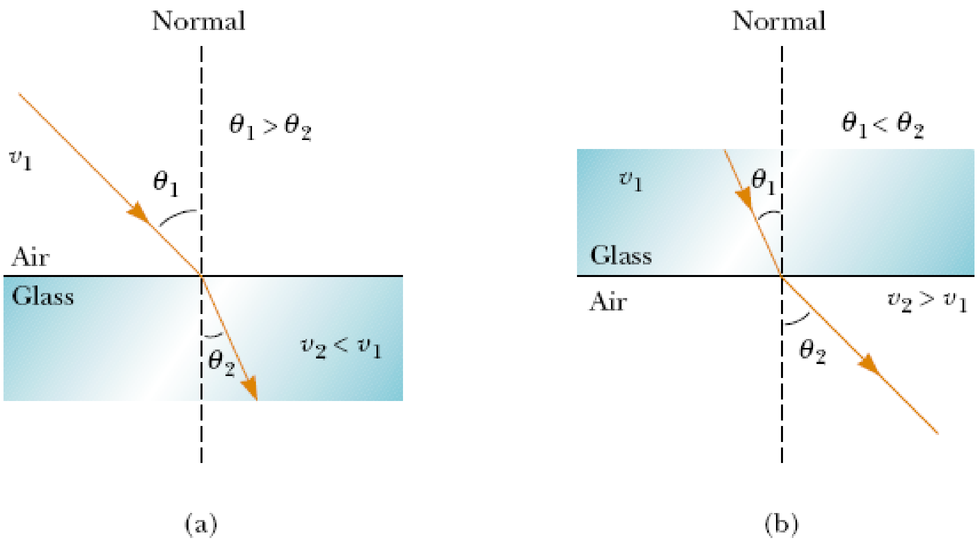


Fig 4.: (a) When the light beam moves from air into glass, the light slows down on entering the glass and its path is bent toward the normal. (b) When the beam moves from glass into air, the light speeds up on entering the air and its path is bent away from the normal.

2.2. REFRACTIVE INDEX

The refraction index of a medium is **defined** as the ratio of velocity of light in a vacuum to the velocity of light in the medium index. Refraction index defined as above is called as absolute refraction index. Thus,

$$\mu = \frac{c}{v} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From this definition, we see that the index of refraction is a dimensionless number greater than unity because v is always less than c. Furthermore, μ is equal to unity for vacuum.

As light travels from one medium to another, its frequency does not change but its wavelength does.

$$v_1 = f\lambda_1 \quad \text{and} \quad v_2 = f\lambda_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$.

We can obtain a relationship between index of refraction and wavelength by dividing the first Equation 4 by the second and then using Equation 3:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/\mu_1}{c/\mu_2} = \frac{\mu_1}{\mu_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This gives

$$\mu_1\lambda_1 = \mu_2\lambda_2$$

If medium 1 is vacuum, or for all practical purposes air, then $\mu_1 = 1$. Hence, it follows from Equation 5 that the index of refraction of any medium can be expressed as the ratio

$$\mu = \frac{\lambda}{\lambda_\mu} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

where λ is the wavelength of light in vacuum and λ_μ is the wavelength of light in the medium whose index of refraction is μ.

We are now in a position to express Equation 2 in an alternative form. If we replace the v_2/v_1 term in Equation 2 with μ_1/μ_2 from Equation 6, we obtain

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

The experimental discovery of this relationship is usually credited to Willebrord Snell(1591–1627) and is therefore known as **Snell's law of refraction**.

Example: A beam of light of wavelength 550 nm traveling in air is incident on a slab of transparent material. The incident beam makes an angle of 40.0° with the normal, and the refracted beam makes an angle of 26.0° with the normal. Find the index of refraction of the material.

Solution:

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

$$\mu_2 = \frac{\mu_1 \sin\theta_1}{\sin\theta_2} = (1.00)\frac{\sin 40.0°}{\sin 26.0°} = \frac{0.643}{0.438} = 1.47$$

2.3. OPTICAL PATH

The shortest distance, L between two point A and B is called the geometric path. The length of geometric path is independent of the medium that surround the path AB. When a light ray travels from the point A to point B, it travels with the velocity (c) if the medium is air and with lesser velocity v if the medium is other than air. Therefore, the light ray takes more time to go from A to B located in a medium.

From equation (3)

$$\mu = \frac{c}{v} = \frac{AB/t}{AB/T} = \frac{T}{t}$$

Where t and T are the time taken by the light ray in air and in a medium respectively.

$$T = \mu t \qquad \text{………………………………………..(8)}$$

The above relation means that a light ray takes μ times more time to cover the distance AB a medium. To take into account the delay, we use another distance called the optical length. If a ray of light travels a distance L in a medium of

refraction index n in a certain interval of time, then it would travel a greater distance Δ in air during same interval of time. Therefore,

$$\frac{\Delta}{L} = \frac{ct}{vt} = \mu$$

or

$$\Delta = \mu L \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

i.e., optical path length =(Refraction index)(Geometric path length)

Thus, **the optical path length is defined as the product of refraction index and the geometric length.**

2.4. DISPERSION

An important property of the index of refraction *n* is that, for a given material, the index varies with the wavelength of the light passing through the material, as Figure 5 shows. This behavior is called **dispersion**. Because *n* is a function of wavelength, Snell's law of refraction indicates that light of different wavelengths is bent at different angles when incident on a refracting material.

The index of refraction generally decreases with increasing wavelength. This means that violet light bends more than red light does when passing into a refracting material. To understand the effects that dispersion can have on light, consider what happens when light strikes a prism, as shown in Figure 6. A ray of single-wavelength light incident on the prism from the left emerges refracted from its original direction of travel by an angle **δ**, called the **angle of deviation**.

Now suppose that a beam of ***white light*** (a combination of all visible wavelengths) is incident on a prism, as illustrated in Figure 6. The rays that emerge spread out in a series of colors known as the visible spectrum. These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet. Clearly, the angle of deviation **δ** depends on wavelength. Violet light deviates the most, red the least, and the remaining colors in the visible spectrum fall between these extremes. Newton

showed that each color has a particular angle of deviation and that the colors can be recombined to form the original white light.

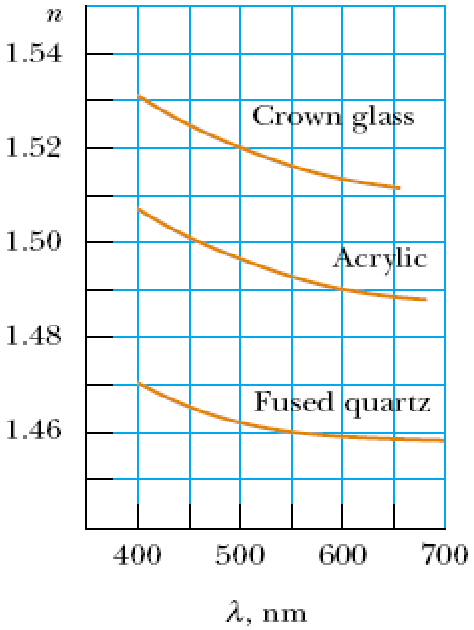


Fig. 5: Variation of index of refraction with vacuum wavelength for three materials.

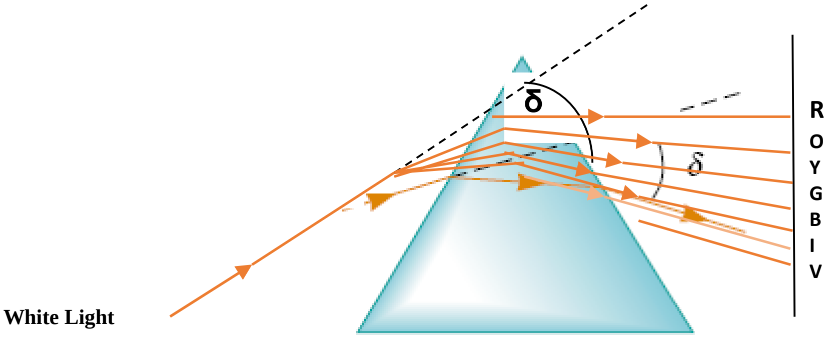


Fig. 6: A prism refracts a single-wavelength light ray through an angle δ.

2.5. THE VELOCITY OF LIGHT

Galileo made the first effort to measure the velocity of light in 1667. Two observers stood on the crests of two hills about 1.5 kilometers apart. Each observer was given one lamp, and the experiment was conducted at night. One witness, say A, uncovered his lamp, sending a brief flash of light to the other observer, say B, and recorded the moment he did so. As soon as he noticed the flare from B's bulb, the second observer B uncovered his lamp. The time spent for light to cover the distance AB twice is plainly equal to the gap between these two instants of time denoted by A.

The gap between the two instants, however, could not be identified. Galileo deduced from this that the velocity of light, if finite, was extraordinarily fast.

In 1675, the Danish astronomer Roemer succeeded in measuring the velocity of light eight years later. Jupiter's satellites show eclipses once every seven days. He noted that the time gap between two successive eclipses of one of Jupiter's satellites changed with the earth's relative position. This time gap grew longer as the earth traveled away from Jupiter and shorter as it got closer to Jupiter. Roemer correctly ascribed this difference to light's limited velocity and discovered that light took approximately 22 minutes to reach a distance equivalent to the circumference of Earth's orbit. The diameter of the Earth's orbit was previously calculated to be 2.87 x1011 m.

$$\mathrm{c} = \frac{2\times 10^{11} m}{22\times 60 sec} = 2.3\times 10^{8}\, m/sec$$

This value is less than the true speed. Regardless of its precision, it demonstrated that the speed of light was not unlimited, as many assumed at the time, but rather bounded and quantifiable.

H.L.Fizeau, a French scientist, performed the first experimental measurement of the speed of light in 1849. Later, several experimenters using various approaches measured c. The most exact calculation yields a value.

$$c = 2.997924 \times 10^{8}\, m/sec \approx 2.3 \times 10^{8}\, m/sec$$

2.6. VISIBLE RANGE

An **electromagnetic spectrum** is the organization of distinct electromagnetic waves in a continuous series of frequencies and wavelengths. The spectrum contains waves with a wide range of wavelengths (see Fig. 7). It is confined on one end by massive radio waves with wavelengths of a few kilometers and on the other by γ-rays with wavelengths of 10^{-12}m. The **visible range** is the portion of the spectrum made up of waves that can be seen with the naked eye. It ranges from deepest violet to deepest red. The limiting range of these waves is determined by the particular features of the eye and ranges about from λ = 4000Å to λ = 7800Å. **Infrared (IR)** on the longer wavelength side and **ultraviolet (UV)** on the shorter wavelength side surround the visible spectrum. The IR region has wavelengths ranging from 7.8 x 10^{-7}m to 10^{-3}m, whereas the UV region has wavelengths ranging from 4000Å to 10Å.

The radiation in these three ranges, namely visible, infrared, and ultraviolet, is referred to as optical radiation. Table 1 shows the wavelengths and frequencies of each coloured area in the visible spectrum.

The sensitivity of the human eye varies with wavelength. Its greatest sensitivity is around 5500 A, which corresponds to yellow-green.

Light wave wavelengths are shorter; hence smaller units are employed to represent them. They are commonly measured in angstroms, nanometers, or micrometers.

TABLE-1

Colour	*Vacuum wavelength (Å)*	*Frequency (10^{14}Hertz)*
Red	**7800 - 6200**	3.84 - 4.82
Orange	**6220 - 5970**	4.82 - 5.03
Yellow	**5970 - 5770**	5.03 - 5.20
Green	**5770 - 4920**	5.20 - 6.10
Blue	**4920 - 455e**	6.10 - 6.59
Violet	**4550 - 3990**	6.59 - 7.69

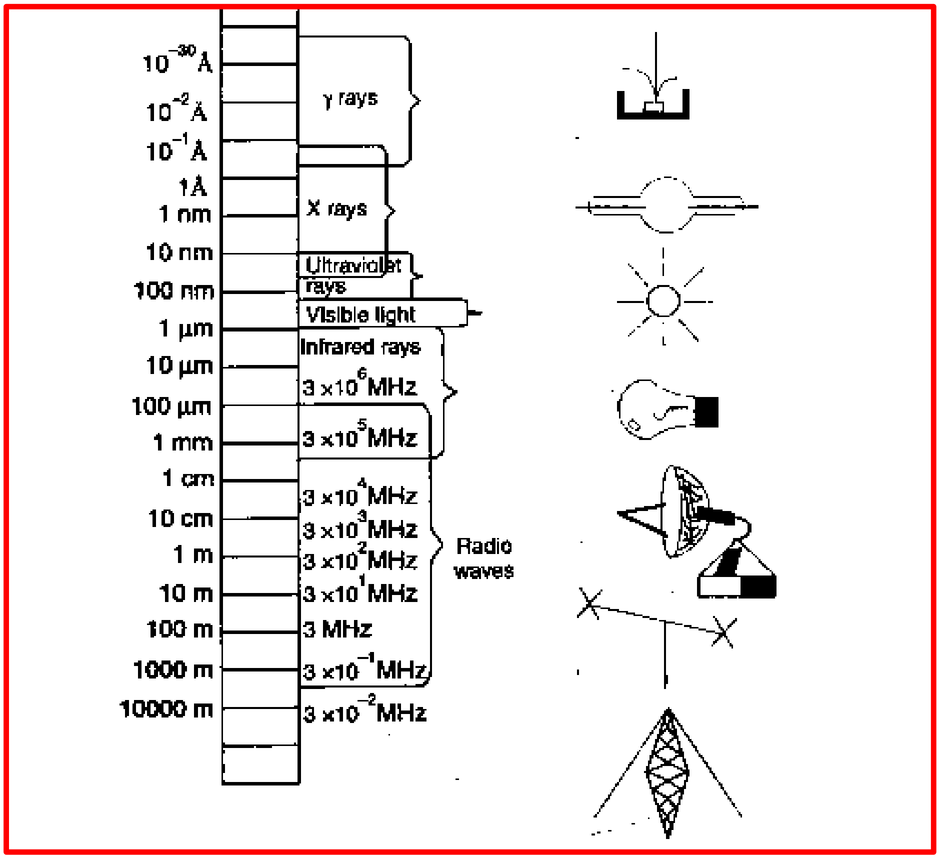


Fig. 7

2.7. PHOTONS

Light, according to quantum theory, is a stream of unique particles known as photons. Photons have no rest mass and move at the speed of light in a vacuum. The essential properties of a photon are its energy E and momentum p.

$$\mathrm{E} = \mathrm{h}\nu \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$\mathrm{E} = \frac{\mathrm{h}\nu}{c} = \hbar k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

A photon's momentum vector p has a direction that coincides with the wave vector k. k has a magnitude $\frac{2\pi}{\lambda}$ and a direction that coincides with the wave velocity. Photon mass $\frac{\mathrm{h}\nu}{c^2}$ is the electromagnetic field's mass and is not related with a rest mass since photons at rest do not exist.

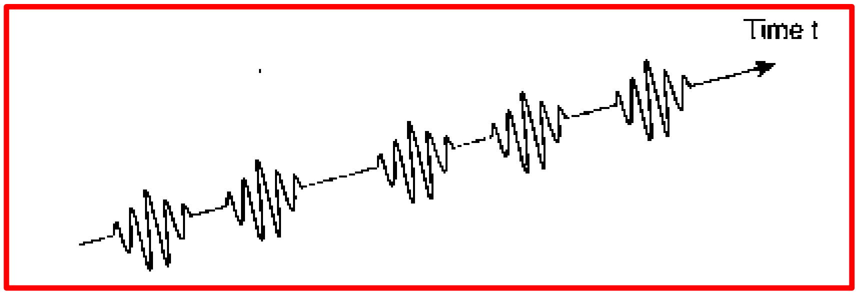


Fig. 8 (a) Bursts of light emitted by atoms.

In light sources, photons are released by individual atoms. When an atom exits an excited state, it releases surplus energy in the form of a burst of light (photon) and returns to the lower normal state. The process of the atom transitioning from a higher to a lower state lasts just around 10^{-8} seconds. As a result, the light emitted by an atom is not a continuous harmonic wave of infinite extension, but rather a wave train of finite length with a finite number of oscillations.

It is difficult to predict when an atom will release light since emission is a fully random process. Figure 8(a) depicts the emission of light by a single atom in terms of wave trains. Other atoms in the source behave similarly, but at differing rates of emission. Adding the wave trains produced by all atoms in the light source results in a sequence of wave trains (Fig. 8 b), giving the impression of a continuous wave.

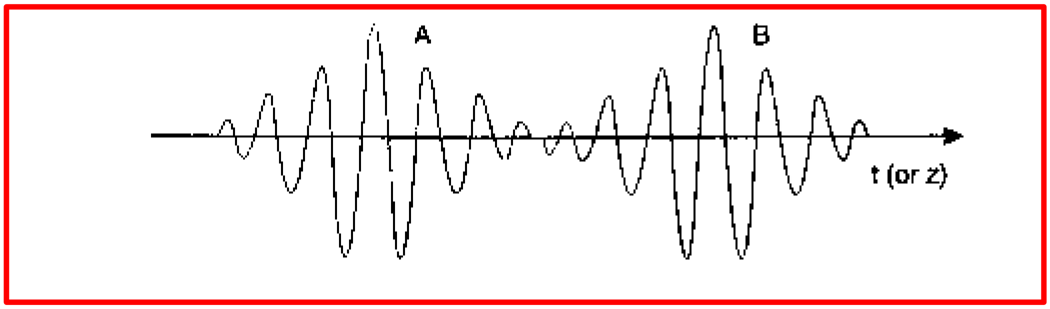


Fig. 8 (b) A succession of wave trains gives an impression of continuous wave.

2.8. THE DUAL NATURE

On the one hand, light seems to be a continuous electromagnetic wave with frequency v, while on the other, it appears to be a collection of photons with energy E and momentum p. It has been discovered that neither model can explain all of the experimental facts individually. A particle is accurately localized in space, but a continuous wave cannot be assigned to a specific point in space. As a result, the corpuscular and wave natures appear to be mutually exclusive. However, experimental data indicates that light acts as both a continuous wave and a particle. As a result, we argue that light has a dual nature.

The explanation for wave-particle dualism may be explained as follows. Radio waves have such wide wavelengths that they spread over a vast amount of space at the lower frequency end of the electromagnetic spectrum (see Fig. 9). As a result, the energy available at any one site is negligible, and their particle nature cannot be detected. We have x- rays and γ-rays on the higher frequency side of the spectrum. Because their wavelengths are so short, the wave energy is concentrated in a very small dimension point, and the particle character is easily seen while the wave nature is less evident. The visible area is a transition region in which both features of light may be seen.

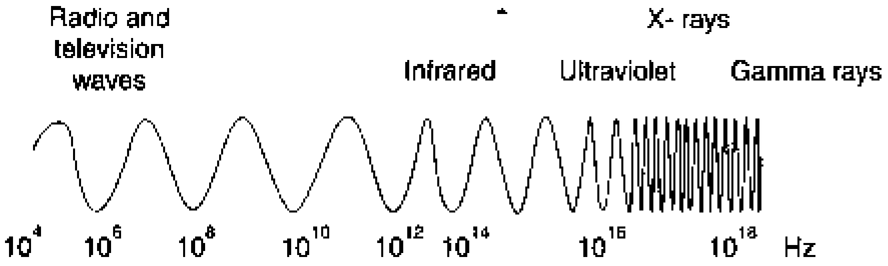


Fig. 9

The photon explanation gives the light intensity as I = Nhν, whereas the wave description gives it as $I = |E|^2$. It indicates that the square of a light wave's amplitude at a location in space is

proportional to the number of photons arriving at that point. In other words, the amplitude of a light wave impacts the likelihood of finding a photon at a certain place in space. The likelihood of observing photons is thus proportional to $|E|^2$. As a result, the ultimate relationship between wave and particle behavior is perceived.

To summarize, light beams are made up of streams of photons, which are essentially electromagnetic wave trains. To explain the behavior of light, we employ all three categories, namely rays, waves, and photons. When light is refracted or reflected by optical devices such as lenses, the ray description is used; when light propagates

across space or any medium, the electromagnetic wave description is used; and when light interacts with matter, the photon description is used.

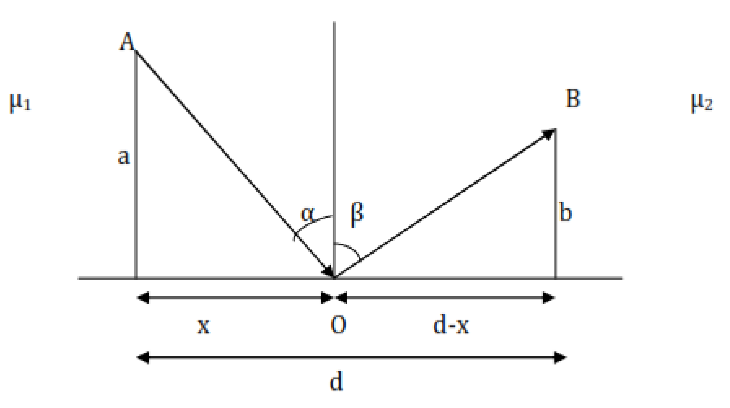
2.9. EERMAT'S PRINCIPLE AND THE REFRACTION LAW

Light has been studied for a long time. Archimedes and other ancient Greek thinkers made original contributions but we mention here Heron of Alexandria (c. 10 - 75 AD) as he was the first to articulate what has come to be known as Fermat's Principle. Fermat, stated his principle as "Light travelling between two points follows a path taking the least time." The modern, and more correct version, is as follows:

> ***" Light must travel along a path in which time is least or distance is Minimum. "***

Fermat's principle is the basis of Geometrical optics which ignores the wave nature of light. The principle may be used to derive Snell's Laws of reflection and refraction.

2.9.1. LAW OF REFLECTION AND REFRACTION FROM EERMAT'S PRINCIPLE:



Optical path $L = \mu_1 AD + \mu_2 OB = \mu_1\left[\sqrt{a^2 + x^2}\right] + \mu_2\left[\sqrt{b^2 + (d - x)^2}\right]$

L=L(x)

$\frac{dL}{dx} = 0$ from minimization principle.

$$\frac{dL}{dx} = \frac{\mu_1}{2}(2x)\left[\left(\sqrt{a^2 + x^2}\right)\right]^{-1} + \frac{\mu_2}{2}\left[2(d - x)(-1)\right]\left[\sqrt{b^2 + (d - x)^2}\right]^{-1} = 0$$

$$\frac{dL}{dx} = \mu_1 \frac{(x)}{\sqrt{a^2+x^2)}} - \mu_2 \frac{(d-x)}{\sqrt{b^2+(d-x)^2}} = 0$$

$$\mu_1 \frac{(x)}{\sqrt{a^2+x^2)}} = \mu_2 \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

$\mu_1 \sin\alpha = \mu_2 \sin\beta$

If medium (1) & (2) is same $\mu_1 = \mu_2$

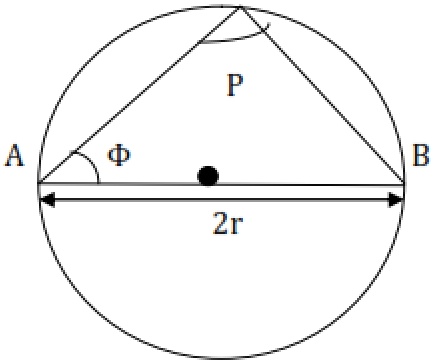
$\sin\alpha = \sin\beta \quad \Rightarrow \quad \alpha = \beta$ **Law of reflection**

If $\mu_1 \neq \mu_2$ medium is different

$\mu_1 \sin\alpha = \mu_2 \sin\beta$ **Snell's Law of refraction**

Problems on optical path& Fermat's principle:

Problem 1 In figure light starts from point A and after reflection from the inner surface of the sphere reaches the diametrically opposite point B. Calculate the length of the hypothetical path APB and using Fermat's principle find the actual path of light. Is the path minimum?



Problem 2: A man walks on the hard ground with a speed of 5 ft. /sec but he has a speed of 3ft. /sec on the sandy ground. Suppose he is standing at the border of sandy and hard ground. The man can reach the tree by walking100 ft. along the border and 120 ft on the sandy ground normal to the border. Find out the value of path which requires minimum time to reach tree.

(a) 190 or 10 (b) 180 or 20 (c) 170 or 30 (d) none

QUESTIONS

1. What is meant by reflection?
2. State and explain the law of reflection.
3. What do you mean by refraction of light?
4. What is absolute refractive index of a medium?
5. What is Snell's law?
6. What is meant by optical path? How is it different from geometrical path length?