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الجامعة المستنصرية
كلية العلوم
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المرحلة ألرابعة

Lecture Title **عنوان المحاضرة**

Measures of Central Tendency ***مقاييس النزعة المركزية***

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The Arithmetic Mean

The arithmetic mean is the average of the data set which is calculated by adding all the data values together and dividing it by the total number of data sets.
Or the arithmetic mean of a set of data is found by taking the sum of the data, and then dividing the sum by the total number of values in the set. A mean is commonly referred to as an average.

The *sample mean* of the values is $X_1, X_2, X_3, \ldots\ldots X_n$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Xi} = \frac{\mathrm{X1, X2, X3, \ldots . Xn}}{n}$$

Example: find the sample mean for (78, 87, 68, 72, 91, 84)

$$\bar{\boldsymbol{X}} = \frac{\mathbf{84+91+72+68+87+78}}{\mathbf{6}} = \frac{\mathbf{480}}{\mathbf{6}} = \mathbf{80}$$

Frequency data: suppose that the frequency of the class with midpoint Xi is fi, for i = 1, 2, ..., m). Then

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{m} \mathrm{f_i} x_i = \frac{\mathrm{f_1} x_1 + \mathrm{f_2} x_2 , + \mathrm{f_3} x_3, \ldots . \mathrm{f_m} x_m}{n}$$

Where n = $\sum_{i=1}^{m}$ $\mathrm{f_i}$ = total number of observations.

Example:

Accidents data: find the sample mean.

Number of accidents, xi	Frequency fi	fi xi
0	55	0
1	14	14
2	5	10
3	2	6

4	0	0
5	2	10
6	1	6
7	0	0
8	1	8
TOTAL	80	54

$$\bar{X} = \frac{54}{80} = 0.675$$

Example 2:

Classes	Fi	Xi	Fi* Xi
31-40	1	35.5	35.5
41-50	2	45.5	91.0
51-60	5	55.5	277.5
61-70	15	65.5	282.5
71-80	25	75.5	1887.5
81-90	20	85.5	1710.0
91-100	12	95.5	1146.0
	∑= 80		∑= 6130

$$\bar{X} = \frac{\sum (f_i x_i)}{\sum (f_i)} = \frac{6130}{80} = 76.62$$

<u>The harmonic mean</u>

is a very specific type of average. It’s generally used when dealing with averages of units, like speed or other rates and ratios.

The formula is:

$$H = \frac{n}{\frac{1}{x^1} + \frac{1}{x^2} + \cdots + \frac{1}{x^n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{xi}}$$

Examples

What is the harmonic mean of 1, 5, 8, 10?

$$\boldsymbol{H = \frac{4}{\frac{1}{1}+\frac{1}{5}+\frac{1}{8}+\frac{1}{10}} = \frac{4}{1.425} = 2.807}$$

Note: the harmonic mean is slightly less than the arithmetic mean.

The geometric mean:

is a type of average , usually used for growth rates, like population growth or interest rates. While the arithmetic mean adds items, the geometric mean multiplies items. Also, you can only get the geometric mean for positive numbers.

$$\text{The geometric mean (GM)} = \sqrt[n]{X1 * X2 * X3 * Xn}$$

Example 1:

What is the geometric mean of 2,3, and 6?

First, multiply the numbers together and then take the cubed root (because there are three numbers) = $(2*3*6)^{1/3} = 3.30$

Example 2:

What is the geometric mean of 4,8.3,9 and 17?

First, multiply the numbers together and then take the 5th root (because there are 5 numbers) = $(4*8*3*9*17)^{(1/5)} = 6.81$

Example 3:
What is the geometric mean of 1/2, 1/4, 1/5, 9/72 and 7/4?

First, multiply the numbers together and then take the 5th root:

(1/2*1/4*1/5*9/72*7/4) $^{\mathbf{(1/5)}}$ = 0.35.

Example 4:
The average person's monthly salary in a certain town jumped from $2,500 to $5,000 over the course of ten years. Using the geometric mean, what is the average yearly increase?

Solution:

Step 1: Find the geometric mean.

(2500*5000)^(1/2) = 3535.53390593.

Step 2: Divide by 10 (to get the average increase over ten years).

3535.53390593 / 10 = 353.53.

The average increase (according to the GM) is 353.53.

The Quadratic Mean (R.M.S.):

The quadratic mean (also called the root mean square*) is a type of average. The quadratic mean is also called the root mean square because it is the square root of the mean of the squares of the numbers in the set.

$$\textbf{R.M.S} = \sqrt{\frac{1}{N}(X_1^2 + X_2^2 + X_3^2 + \cdots + X_n^2)}$$

Example:

Find the Quadratic Mean (R.M.S.) for the following data: (3,5,6,6,7,10,12)

$$R.M.S. = \sqrt{\frac{\sum(\bar{X})^2}{N}} = \sqrt{\frac{(3)^2+(5)^2+(6)^2+(6)^2+(7)^2+(10)^2+(12)^2}{7}}$$
$$= \sqrt{57} = 7.55$$

The Quadratic Mean (R.M.S.) for grouped data:

$$R.M.S. = \sqrt{\frac{\sum(Xi)^2 x\, fi}{\sum fi}}$$

Example: Find the Quadratic Mean (R.M.S.) for the following data:

Classes	Frequency fi	Mid-Point Xi
60 – 62	5	61
63 – 65	18	64
66 – 68	42	67
69 – 71	27	70
72 - 74	8	73

The solution :

Classes	Frequency fi	Mid-Point Xi	$(Xi)^2$	fi x $(Xi)^2$
60 – 62	5	61	3721	18605
63 – 65	18	64	4096	73728
66 – 68	42	67	4489	188538
69 – 71	27	70	4900	132300
72 - 74	8	73	5329	42632
	∑ 100			∑455803

$$R.M.S. = \sqrt{\frac{\sum(Xi)^2 x\, fi}{\sum fi}} = \sqrt{\frac{455803}{100}} = 67.51$$

weighted mean

A **weighted mean** is a kind of average. Instead of each data point contributing equally to the final mean, some data points contribute more "weight" than others. If all the weights are equal, then the weighted mean equals the arithmetic mean (the regular "average" you're used to). Weighted means are very common in statistics, especially when studying populations.

The formula can be written as:

$$\text{Weighted mean} = \frac{\Sigma wi.xi}{\Sigma wi}$$

- Σ = summation (in other words…add them up!).
- w = the weights.
- x = the value.

Example: - The following data represent the scores of a student in seven subjects and the units of these subjects, find the **weighted mean**

The scores (x): 61,63,56,68,72,70,66
The units (w): 2, 3, 2, 3, 2, 3, 2

$$\text{Weighted mean} = \frac{\Sigma wi.xi}{\Sigma wi}$$

$$= \frac{(61 \times 2 + 63 \times 3 + 56 \times 2 + 68 \times 3 + 72 \times 2 + 70 \times 3 + 66 \times 2)}{17}$$

weighted mean = 65.47

weighted mean for grouped data:

$$\text{Weighted mean} = \frac{\Sigma \, \text{wi}.fi.\text{xi}}{\Sigma \, \text{wi}.\text{fi}}$$

Example:
find the weighted mean for the following data:

Classes	Frequencies	Wi
0-2	2	5
2-4	3	6
4-6	6	4
6-8	4	5
8-10	1	4
Total	16	

Solution:

Classes	Frequencies	Wi	Xi	Wi* Fi	Wi* Fi*Xi
0-2	2	5	1	10	10
2-4	3	6	3	18	54
4-6	6	4	5	24	120
6-8	4	5	7	20	140
8-10	1	4	9	4	36
Total	***16***			***76***	***360***

$$\text{Weighted mean} = \frac{\Sigma \text{wi}.fi.\text{xi}}{\Sigma \text{wifi}}$$

$$\text{Weighted mean} = \frac{360}{76} = 4.7$$

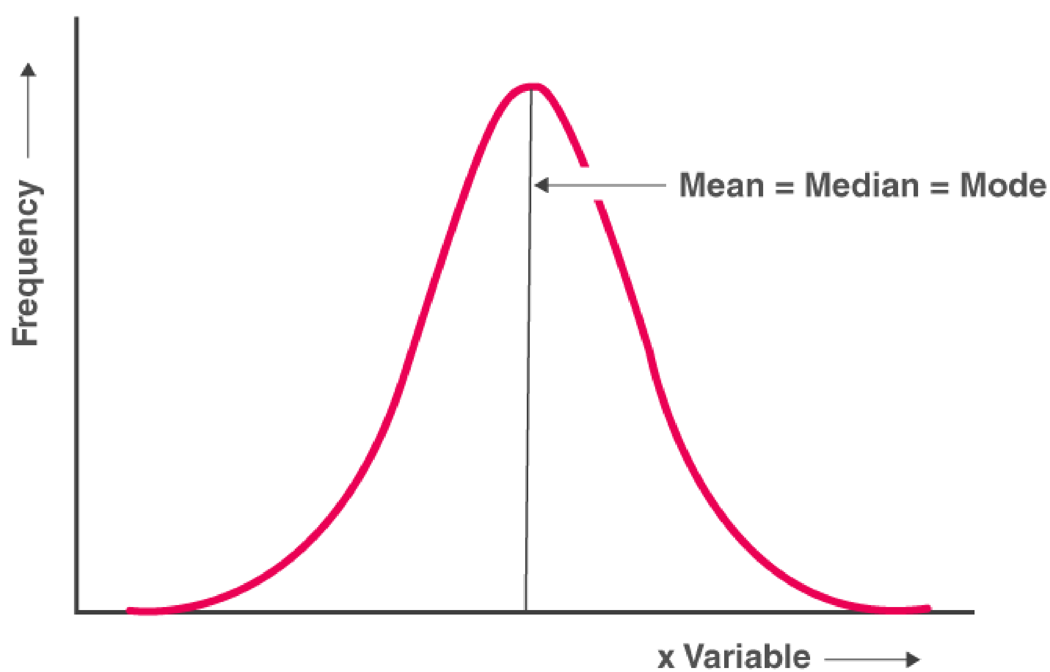
The relationship between the mean, the median, and the mode

In statistics, for a moderately skewed distribution, there exists a relation between mean, median and mode. This mean median and mode relationship is known as the "empirical relationship" which has been discussed in detail below.

$$\textit{Mean} - \textit{Mode} = 3\,(\textit{Mean} - \textit{Median})$$

Mean Median Mode Relation with Frequency Distribution

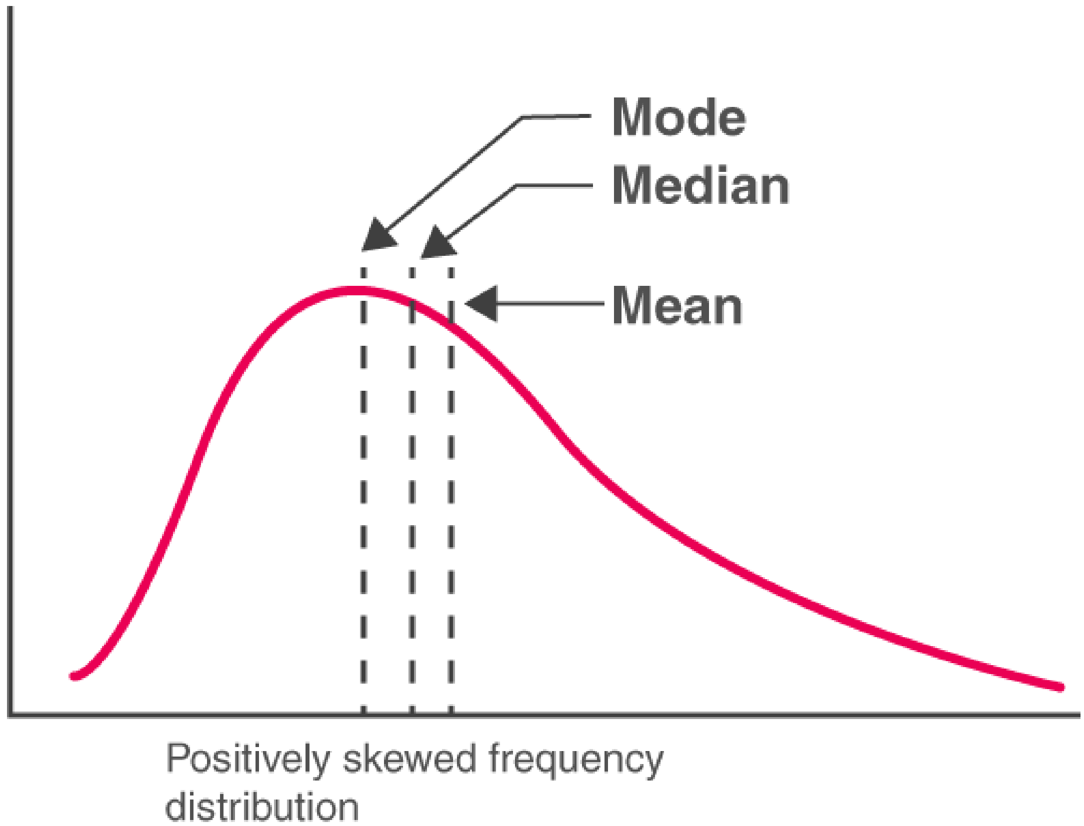
1- If a frequency distribution graph has a symmetrical frequency curve, then mean, median and mode will be equal.



Mean = Median = Mode

2- For Positively Skewed Frequency Distribution

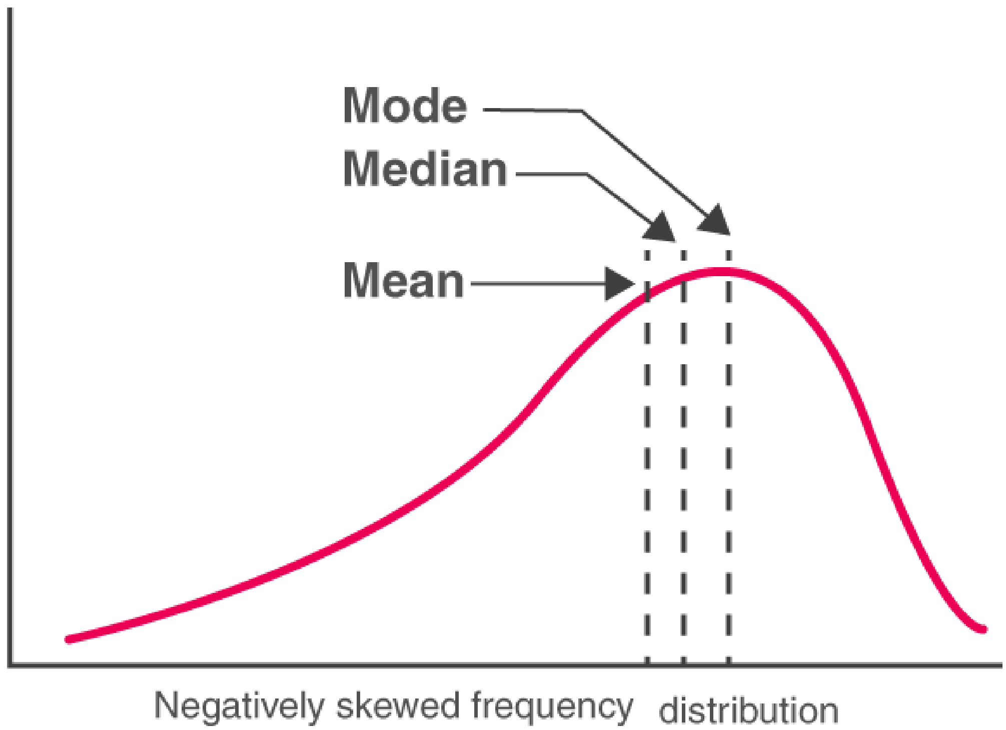
In case of a positively skewed frequency distribution, the mean is always greater than median and the median is always greater than the mode.



Mean > Median > Mode

3- For Negatively Skewed Frequency Distribution

In case of a negatively skewed frequency distribution, the mean is always lesser than median and the median is always lesser than the mode



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Mean < Median < Mode

Question: In a moderately skewed distribution, the median is 20 and the mean is 22.5. Using these values, find the approximate value of the mode. Determine the type of skewed distribution.

Solution:

Given,

Mean = 22.5

Median = 20

Mode = x

Now, using the relationship between mean, mode and median we get, (Mean – Mode) = 3 (Mean – Median)

So,

22.5 – x = 3 (22.5 – 20)

22.5 – x = 7.5

$\therefore$ x = 15

So, Mode = 15.

Mean > Median > Mode

22.5 **> 20 >** 15 positive skewed.