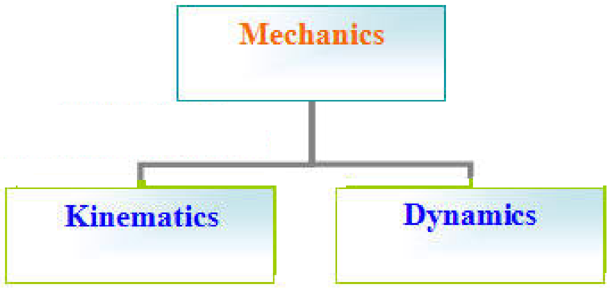
3- Mechanics

The oldest one of the Physics subjects is the Mechanics. Is one of the broad sciences concerned with the movement of bodies and their causes. The Mechanics can be divided into two parts: Kinematics and Dynamics.



3-1 Kinematic

The kinematics aims the motion. The science of kinematics is concerned with describing the movement of bodies without regard to its causes. It answers the questions such that: Where the motion started? Where the motion stopped? What time has taken for the complete of motion? What the velocity body had?

3-2 Dynamic

The dynamics deals with the effects that create the motion or change the motion or stop the motion. It takes into account the forces and the properties of the body that can affect the motion, it studies the movement of objects and their causes, such as force and mass.

3- 4 The position vector and the displacement vector

One of the basics of studying the science of describing the kinematics of physical bodies is the study of ***displacement***, ***velocity***, and ***acceleration***. Here, we need to adopt coordinate axes to determine the position of the moving body at different times. It is appropriate to adopt the Cartesian coordinate axes, or what is called rectangular coordinate (x, y, z). For example, we need to determine the location of an object to assign it to a specific reference.

As in Figure 3.1, where the reference centre in two dimensions was considered only as the centre of the x, y axes.

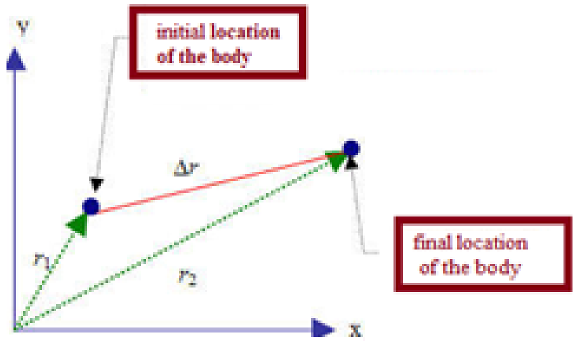


figure 3.1 position body

We first need some definitions to identify the motion. It begins by defining the change in position of a particle. We call it "displacement". Displacement is defined to be the change in position or distance that an object has moved. In Figure 3.1, the position vector $\boldsymbol{r}_1$ determines the position of the body at the beginning of the movement, and the position vector $\boldsymbol{r}_2$ determines the position of the final body after a time of magnitude $\Delta \mathbf{t} = \mathbf{t}_2 - \mathbf{t}_1$. Here, the displacement of the body is given by the equation:

$\mathbf{r}_1 = x_1\boldsymbol{i} + y_2\boldsymbol{j}$

$\mathbf{r}_2 = x_2\boldsymbol{i} + y_2\boldsymbol{j}$

$\Delta\mathbf{r} = \vec{r}_2 - \vec{r}_1$

Where:

Δr is the displacement vector which represent the change in the position vector.

$\vec{r}_2$ is the final position and $\vec{r}_1$ is the initial position. The arrow indicates that displacement is a vector quantity: it has direction and magnitude, as we mentioned before**.** In fact, $\vec{r}$ is in 3-dimension and written as**:**

$\vec{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$

Displacement Δr: depends only on the distance between the start and end points and <u>does not depend on the path taken by the object</u>

It is not enough to define the displacement for a motion. In order to be useful, we also need to specify something about ***time***. In the language of mathematics, we describe the changes in position, $\Delta x = \vec{x}_2 - \vec{x}_1$ and time as: $\Delta t = \mathbf{t}_2 - \mathbf{t}_1$

Where the ***i*** and ***f*** subscripts depict <u>**initial**</u> and <u>**final**</u>, respectively.

Generally, $t_i = \mathbf{t}_0 = \mathbf{0}$

3-5 The *average velocity* and *Instantaneous velocity*:

To quantify the amount of motion taking place, we should define the average velocity or speed of an object. The idea of quantifying motion involves both the distance travelled by the body and the time the body took to travel it.

When the body moves from the starting position at time $\mathbf{t_1}$ to the end position at time $\mathbf{t_2}$, the displacement product divided by the time difference $\Delta t = t_2 - t_1$ is known as velocity, and since the body travels the distance at different speeds, the calculated velocity is called *average velocity*. The velocity at any moment can be defined as *instantaneous velocity*.

The *average velocity* of a particle is defined as the ratio of the displacement to the time interval

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{x - x_0}{t - t_0}$$ The unit of the velocity is (**m/s**)

The *instantaneous velocity* of a particle is defined as the limit of the average velocity as the time interval approaches **zero.**

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{dx}{dt}$$

This is the instantaneous velocity of the particle and its magnitude is called speed.

3-6 The *average acceleration* and *Instantaneous acceleration*:

When the body moves from the starting position at time $\mathbf{t_1}$ to the ending position at time $\mathbf{t_2}$ with an initial velocity $\mathbf{v_1}$ and at the end the velocity is $\mathbf{v_2}$, the rate of change of velocity with respect to time is known as **acceleration $\vec{a}$** or **average acceleration**.

$$\vec{a}_{av} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{v - v_0}{t - t_0}$$ Average acceleration is the change in velocity over the change in time

The direction of the acceleration is in the direction of the vector Δv, and its magnitude is $|\Delta v / \Delta t|$.

The *instantaneous acceleration* is defined as the limiting value of the ratio of the average velocity to the time interval as the time approaches zero.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{dv}{dt}$$ The unit of the acceleration is (**m/s²**)

It should be noted that:

Acceleration is the rate of change of velocity.

When velocity and acceleration are in the same direction, speed increases with time.

When velocity and acceleration are in opposite directions, speed decreases with time.

Graphical interpretation of acceleration: On a graph of ***v*** *versus* ***t****, the average acceleration between A and B is the slope of the line between A and B, and the instantaneous acceleration at A is the tangent to the curve at A.*

3-7 The Motion

3- 7-1 Motion in One Dimension with *constant acceleration:*

As it is understood from the subtitle, constant acceleration means velocity increases or decreases at the same rate throughout the motion. Example: an object falling near the earth's surface (neglecting air resistance).

3-7-2 Derivation of Kinematics Equations of Motion:

We choose: $t_i = t_0 = 0$, $x_i = x_0$, $v_i = v_0$ and $t_f = t$, $x_f = x$, $v_f = v$ .Since $\vec{a}$ constant, then $\vec{a} = a$.

$$\vec{\boldsymbol{v}} = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{x - x_0}{t - t_0} \text{ average velocity}$$

$$\vec{\boldsymbol{v}} = \frac{dx}{dt}$$

$$dx = \vec{\boldsymbol{v}}\, dt$$

$$\int_{x_0}^{x} dx = \int_0^t \vec{\boldsymbol{v}}\, dt$$

$\Rightarrow$ $\mathbf{x = x_0 + vt}$ ------------------------------- ----- (equation **1**)

In the same way

$$\vec{\boldsymbol{a}} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{v - v_0}{t - t_0}$$

$$\vec{\boldsymbol{a}} = \frac{dv}{dt}$$

$\Rightarrow$ $\mathbf{v = v_0 + at}$ ----------------------------------(equation **2**)

Show that **$V^2 = V_0^2 + 2ax$ -------------------------------**(equation **3**)

Answer:

$$\vec{a} = \frac{dv}{dt} \cdot \frac{dx}{dx} \quad = \frac{dx}{dt} \cdot \frac{dv}{dx} = v \cdot \frac{dv}{dx}$$

$$\Rightarrow \int_{v_0}^{v} \mathrm{v}\, dv = \int_{x_0}^{x} \mathrm{a}\, dx$$

$$\frac{v^2}{2} - \frac{{v_0}^2}{2} = a\,(x - x_0)$$

$(\boldsymbol{v^2 = v_0{}^2 + 2ax})$ *It should be noted that this equation does not depend on time, **t**.*

From equations **2** & **3** we get:

$$\boldsymbol{x = v_0\, t + \frac{1}{2} at^2}$$ **---** Show that? **---------**(equation **4**)

Q1) A body moves along the x-axis according to the following relationship:

$x = 2t^3 + 5t^2 + 5$ where x in meter(m) and t in second (s)

Find:1- Speed and acceleration at any time

2- Position, speed and acceleration after 2 seconds and 3 seconds

3- The rate of speed and acceleration between these two time periods

Answer:

1- $\vec{v} = \frac{dx}{dt} = 6t^2 + 10t$

$\vec{a} = \frac{dv}{dt} \quad = 12\,t + 10$

2- When t=2:

$x = 2(2)^3 + 5(2)^2 + 5 = 41$ m

$v = 6(2)^2 + 10\,(2) \quad = 44$ m s^{-1}

$a = 12(2) + 10 \quad = 34$ m s^{-2}

3- At, t=3 sec home work $\vec{v} = \frac{\overrightarrow{\Delta x}}{\Delta t}$, $\vec{a} = \frac{\overrightarrow{\Delta v}}{\Delta t}$

Home work:

Q1): show that, we can get equation 3 from sub equations 2 & 4