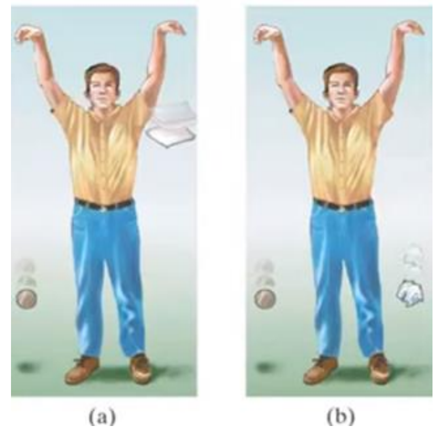
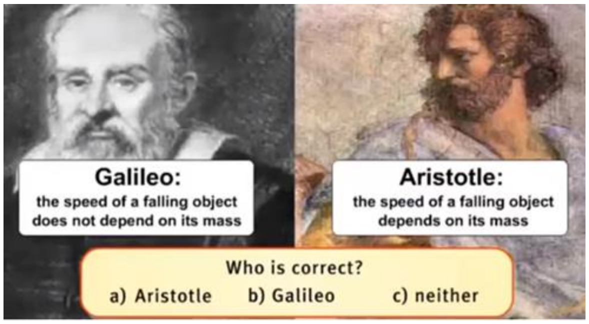
Freely Falling Bodies

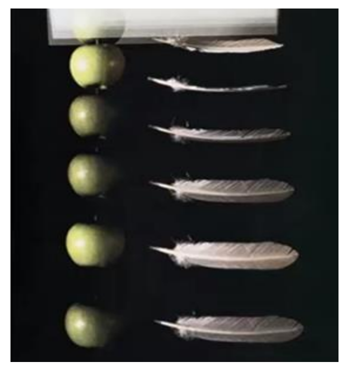
Near the surface of the earth, all objects experience approximately the same acceleration. In the absence of are resistance, all objects fall with the same acceleration, although this may be hard to tell by testing in an environment where there is air resistant.





Free fall acceleration

One important example of constant acceleration is 'free fall' of an object under the influence of the Earth's gravity. the picture shows an apple and feather falling in vacuum with identical motion.



- Earth's gravity provides an acceleration, most important case of constant acceleration.
- Free - fall acceleration is independent of mass.
- Magnitude: $|a| = g = 9.81\ \text{m/s}^2$
- Direction: always downward, so a_g is negative if defined 'up' as positive:
  $a = -g = -9.8\ \text{m/s}^2$

The free fall equations are the same as the motion equations for the body, but they differ from them only because they depend on gravity. Which whose movement is always towards the bottom.

If motion is straight up and down and we can choose a coordinate system with the positive y-axis pointing up and perpendicular to the earth's surface, then we can describe the motion with Equations (1,2,3,4) with $\boldsymbol{a} \rightarrow -\boldsymbol{g}, \boldsymbol{x} \rightarrow \boldsymbol{y}$. (Negative sign arises because the coordinate system is changed and the acceleration direction is downward.)

So that, equations of motion for the 1-dimensional vertical motion of an object in free-fall can be written as following:

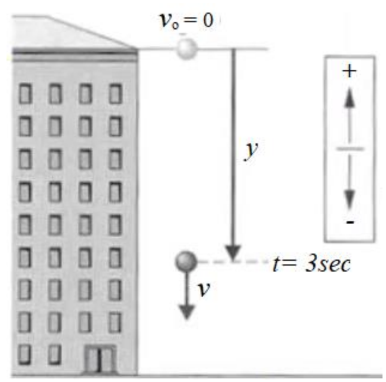
$$v = v_0 + at \longrightarrow v = v_0 - gt$$

$$x = x_o + \frac{1}{2}(v + v_o)t \longrightarrow y = y_o + \frac{1}{2}(v + v_o)t$$

$$x = x_o + v_o t + \tfrac{1}{2}at^2 \longrightarrow y = y_o + v_o t - \tfrac{1}{2}gt^2$$

$$v^2 = v_0^{\ 2} + 2a(x - x_o) \longrightarrow v^2 = v_0^{\ 2} - 2g(y - y_o)$$

Q1) A stone is dropped from rest from the top of a building. After 3s of free fall, what is the *displacement y* of the stone?



From equation

$$y = y_o + v_o\, t - \frac{1}{2}\, g\, t^2$$

$$y = 0 + 0 - (9.8) \times (3)^2 = -44.1\text{m}$$

Find the speed of the stone after 3 seconds.

Q2) A student throws a set of keys vertically upward to another student in a window 4m above as shown in Figure. The keys are caught 1.5s later by the student.

(a) With what initial velocity were the keys thrown?

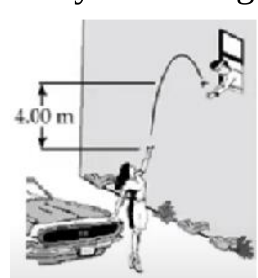
(b) What was the velocity of the keys just before they were caught?

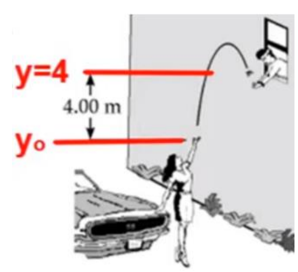
(a) Let $y_o = 0$ and y =4 m at t = 1.5sec then we find

$$y = y_o + v_o\, t - \frac{1}{2}\, g\, t^2$$

$$4 = 0 + 1.5\, v_o - 4.9\, (1.5)^2$$

$$v_o = 10 \text{ m/s}$$





(b) The velocity at any time $t > 0$ is given by

$v = v_o - gt$

$v = 10 - 9.8\, (1.5) = -\, 4.68$ m/s (*What does the negative sign mean*)

Q3) A body moves along the x-axis according to the following relationship:

$x= 2t^3 +5t^2+ 5$ where x in meter(m) and t in second (s)

Find:
1- Speed and acceleration at any time
2- Position, speed and acceleration after 2 seconds and 3 seconds
3- The rate of speed and acceleration between these two time periods

Answer:

1- $\vec{v} = \frac{dx}{dt} = 6t^2 +10t$

$\vec{a} = \frac{dv}{dt} = 12\,t +10$

2- When t=2:

$x = 2(2)^3 + 5(2)^2 + 5= 41$ m

$v= 6(2)^2 + 10\,(2) = 44$ m s^{-1}

$a =12(2) + 10 = 34$ m s^{-2}

At, t=3 sec

$x = 2(3)^3 + 5(3)^2 +5= 104$m

$v= 6(3)^2 + 10(3) = 84$ m/ s

$a= 12(3) +10 = 46$ m/s^2

3- $\vec{v} = \frac{\overrightarrow{\Delta x}}{\Delta t} = \frac{104-41}{3-2} = \frac{63}{1} = 63$m/s

4- $\vec{a} = \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{84-44}{3-2} = \frac{40}{1} = 40$ m/ s^2

Q4) Assume that a car decelerates at 2.0m/s^2 and comes to a stop after traveling 25m.
a) Find the speed of the car at the start of the deceleration.
b) Find the time required to come to a stop.

Solution:

We are given: a =-2 m/s^2
x = 25 m
v =?

a) From:

$v^2 = v_0^2 + 2ax \Rightarrow v_0^2 = v^2 - 2ax \Rightarrow v_0^2 = 0 -2\,(-2)\,(25) = 10\ m$ /s

b) From: $v= v_0 + at$ we have: $t = \frac{v-v_0}{a} = \frac{0-10}{2} = 5\ sec$

Q5) Assume that a car traveling at a constant speed of 30 m/s passes a police car at rest. The policeman starts to move at the moment the speeder passes his car and accelerates at a constant rate of 3 m/s^2 until he pulls even with the speeding car.

a) Find the time required for the policeman to catch the speeder and
b) Find the distance traveled during the chase.

Solution:

We are given, for the speeder:
$v^s{}_0$= 30 m/s, *constant speed*, then a^s =0

and for the policeman:
$a_0{}^p$ = 3 m/s^2

a) The distance traveled by the speeder is given as $x^s = v^s t = 30t$.

Distance traveled by policeman $x^p = x_0{}^p + v_0{}^p t + \frac{1}{2} a^p t^2$

When the policeman catches the speeder $x^s = x^p \Rightarrow 30t = 0+0+\frac{3t^2}{2}$

Solving for t we have t= 0 and t= *20sec*.
The first solution tells us that the speeder and the policeman started at the same point at t = 0, and the second one tells us that it takes 20sec for the policeman to catch up to the speeder.

b) Substituting back in above we find the distance that the speeder has taken

x^s= 30(20) = 600 m.
And also, for the policeman

$$x^p = x_0{}^p + v_0{}^p t + \frac{1}{2} a^p t^2 \quad = 0 + 0 + \frac{1}{2}(3)(20)^2 = 600\text{m}$$

Q6) A rocket moves upward, starting from rest with an acceleration of 29.4m/s^2 for 4 sec. At the end of this time, it runs out of fuel and continues to move upward. How high does it go totally?

Solution:

For the first stage of the flight, we are given:

a = 29.4 m/s^2 for t = 4sec

This gives us the velocity and position at the end of the first stage of the flight:

$v = v_0 + at$ = 0+29.4(4) = 117.6 m/s

and $y = v_0 t + \frac{1}{2} a t^2 = 0 + 0 + \frac{1}{2} 29.4(4)^2 = 235.2$ m

For the second stage of the flight, the rocket will go upward with its velocity till it stops (That is the Newton's Motion Law). So, we start with

$v_o = 117.6$ m/s & $a = g = -9.8$ m/s^2

And we end up with $v^2 = 0$. We want to find the distance traveled in the second stage $(y_2 - y_1)$. We have

$$v^2 = v_0^2 - 2g\,(\Delta y) \quad \Rightarrow \Delta y = y_2 - y_1 = \frac{v^2 - v_0^2}{-2g} = \frac{0-(117.6)^2}{-2(9.8)} = 705.6 \text{ m}$$

Therefore, the total distance taken by the rocket is: $y_2 = y_1 + 705.6 = 940.8$m

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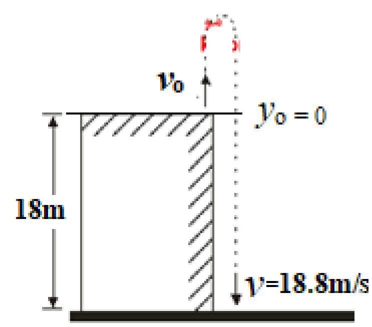
Q7) A stone is thrown upwards from the edge of a cliff 18m high as shown in Figure. It just misses the cliff on the way down and hits the ground below with a speed of 18.8m/s.

(a) With what velocity was it released?

(b) What is its maximum distance from the ground during its flight?

Solution

Let $y_o = 0$ at the top of the cliff.



(a) From equation

$$v^2 = v_o^2 - 2g\,(y - y_o)$$

$$(18.8)^2 = v_o^2 - 2 \times 9.8 \times 18$$

$$v_o^2 = 0.8 \text{ m/s}$$

(b) The maximum height reached by the stone is y_{max}:

$$y_{max} = \frac{v2}{2g} = \frac{18.8}{2x\ 9.8} \cong 18 \text{ m}$$

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from $v = v_o - gt \quad \Rightarrow \quad t = \frac{vo}{g}$ sub. This equation in $y = y_o + v_o\, t - \frac{1}{2} g\, t^2$ we gat

$y = y_o + \frac{vo2}{2g}$ show that?