Projectile Motion

As we know well, the projectile motion is a particular kind of 2-dimensional motion. Firstly, we will make the following assumptions: The only force present is the force due to gravity. The magnitude of the acceleration due to gravity is ‖g‖ = g= 9.8m/s². We choose a coordinate system in which the positive y-axis points up perpendicular to the earth's surface. This definition gives us that $\overrightarrow{a_y} = -g\,j = -9.8\ m/s^2$ and $\overrightarrow{a_x} = 0$.

Initial Conditions:

We choose the coordinate system, so that the particle leaves the origin (x_0 =0, y_0 = 0) at time t_i = 0, with an initial velocity of v_i. The Procedure for Solving Projectile Motion Problems are as followings:

1. We will separate the motion into the x (horizontal) part and y (vertical) part.
2. Then we will consider each part separately using the appropriate equations. The equations of motion, for each component, become:
   a. x-motion (a_x =0);

Example: A bullet is fired from a rifle at a speed of 200 m/s at an angle of 40° with horizon, Find:
1- The speed and position of the bullet after 20 seconds.
2- The range and flight time of the bullet.
Solution:

1- v_o = 200 m/s, θ = 40°,

$$v_{xo} = v_o \cos\theta_o = 200 \cos 40 = 153.2 \text{ m/s}$$

$$v_{yo} = v_o \sin\theta_o = 200 \sin 40 = 128.6 \text{ m/s}$$

$$v_x = v_{xo} = v_o \cos\theta_o = 153.2 \text{ m/s}$$

$$v_y = v_{yo} - gt = 128.6 - 9.8 \times 20 = -67.4 \text{ m/s}$$

$$v = \sqrt{vx^2 + vy^2} = \sqrt{153.2^2 + (-67.4)^2} = 167.4 \text{ m/s}$$

$$x = v_{xo}\, t = v_o\, t \cos\theta_o = 153.2\, t = 3064 \text{ m}$$

$$y = v_{yo}\, t - \frac{1}{2} g\, t^2 = v_o\, t \sin\theta_o - \frac{1}{2} g\, t^2 = 128.6\text{x}20 - \frac{1}{2}\, 9.8\text{x}400 = 612 \text{ m}$$

$$T = \frac{2v_o \sin\theta_o}{g} = \frac{2x200sin40}{9.8} = 26.24 \text{ sec}$$

$$R = \frac{v_o^2\, sin2\,\theta_0}{g} = \frac{200^2\ sin2(40)}{9.8} = 4021\text{m}$$

Q1) A body is thrown at an angle of 30° to the horizontal with an initial velocity of 8 m/s. Find

1- The time the body reaches its highest point
2- The maximum height reached by the body
3- The time the body stays in the air
4- The horizontal range.

Solution:

1- $t = \frac{v_{y_0} \sin\theta_0}{g} = \frac{8sin30}{9.8} = 0.4$ sec

2- $y=h = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{8^2 \ \sin^2 30}{2x9.8} = 0.82$ m

3- $T = \frac{2v_0 \sin\theta_0}{g} = \frac{2x8 \ sin30}{9.8} = 0.81$sec

4- **R** $= \frac{8^2 \ \ sin2 \ x \ 30}{9.8} = 5.65$ m

Q2) A projectile is fired at a speed of 600 m/s at angle of 60° with the horizontal, calculate:

1- The horizontal range
2- The maximum height
3- Speed and altitude after 30 seconds
4- The speed and time of the projectile at an altitude of 10 km.

Solution: $v_0 = 600$ m/s, $\theta = 60°$

1- **R** $= \frac{v_0^2 \ sin2\,\theta_0}{g} = \frac{600^2 \ sin2\,x60}{9.8} = 31800$ m = 31.6 km

2- $h = \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{600^2(sin60)^2}{2x9.8} = 13800$ m = 13.8 km

3- $v_x = v_{xo} = v_o \cos\theta_o$
= 600 cos60 = 300 m/s

$v_y = v_{yo} - gt = v_o \sin\theta_o - gt$
= 600 sin 60 – 9.8 x 30 = 225.6 m/s

$v = \sqrt{vx^2 + vy^2} = \sqrt{31800^2 + 225.6^2}$

4- y= 10km = 10000 m

$$\mathbf{y} = v_{yo}\, t - \frac{1}{2} g\, t^2 = v_o\, t \sin\theta_o - \frac{1}{2} g\, t^2$$

$$10000 = 600 \sin 60 \times t - \frac{1}{2} 9.8\, t^2 \Rightarrow t = 25 \text{ sec}$$

$v_x = v_{xo} = v_o \cos\theta_o$ = 600 cos 60 =300 m

$v_y = v_{yo} - gt = v_o \sin\theta_o - gt$

= 600sin60 – 9.8 x 25 = 274.6 m

$\mathbf{v} = \sqrt{vx^2 + vy^2} = \sqrt{300^2 + 274.6^2}$ = 406.7 m