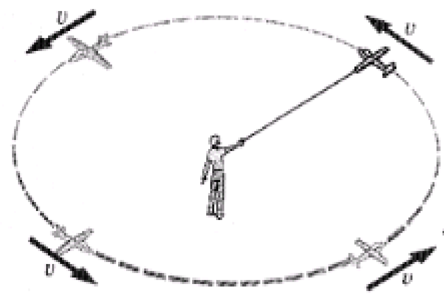
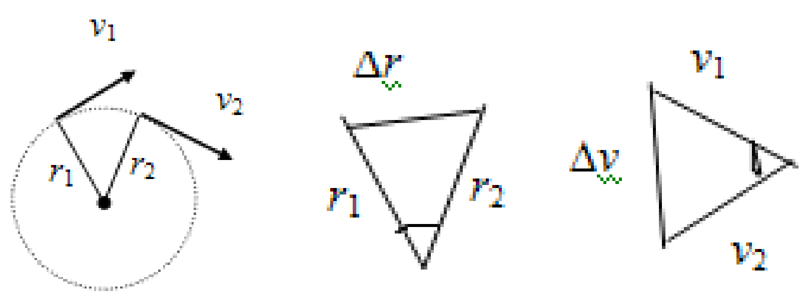
6-1 Motion in Uniform Circular Motion

A particle can move along a circular path with a constant linear speed.



It may occur to us know that, the acceleration in this case is equal to zero, because the speed is constant, and this is incorrect because the body moves on a circular path, so there is acceleration. To explain this, we know that velocity is a vector quantity, and acceleration is a vector quantity because it is equal to the rate of change in velocity with respect to time, and the change in velocity may be in magnitude or in direction.

In the case of the movement of the body on a circular path, the acceleration does not affect the amount of speed, but it is changing the direction of the speed, and for this reason the body moves on a circular path at a constant speed. The velocity vector is always perpendicular to the radius and in the direction of the tangent at any point on the circular path as in the figure.



$$\frac{\Delta v}{\Delta r} = \frac{v}{r}$$

$\Rightarrow \Delta v = \frac{v}{r}\,\Delta r$ Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t}$$

$a = \frac{v}{r}\,v = \frac{v^2}{r}$ where ; $a_\perp$: vertical acceleration, *r:* radius,

$\boxed{\therefore \boldsymbol{a}_\perp = \frac{v^2}{r}}$ and the tangent acceleration $\boxed{a_t = \frac{\mathrm{d}v}{\mathrm{d}t}}$

$\boldsymbol{a} = \sqrt{a_{\perp}^2 + a_t^2}$ and $\theta = \tan^{-1} \dfrac{\boldsymbol{at}}{\boldsymbol{a}_{\perp}}$

Note:

If the speed $\boldsymbol{v}$ is constant, then the tangential acceleration $\boldsymbol{a_t} = 0$ and the perpendicular acceleration $\boldsymbol{a}_{\perp} \neq 0$ and it is perpendicular to the concave side.

Example1:

A particle moves in a circular path 0.4m in radius with constant speed. If the particle makes five revolutions in each second of its motion, find:

(a) The speed of the particle and (b) its acceleration.

Solution

(a) Since r=0.4m, the particle travels a distance 0f $2\pi r = 2.51$m in each revolution. Therefore, it travels a distance of 12.55m in each second (since it makes 5 rev. in the second).

$\therefore \boldsymbol{v} = 12.55\text{m}/1\text{sec} = 12.5 \text{ m/s}$

(b) $a_{\perp} = \dfrac{v^2}{r} = \dfrac{12.55^2}{0.4} = 393\text{m/s}^2$

Example2:

A train slows down as it rounds a sharp horizontal turn, slowing from 90km/h to 50km/h in the 15s that it takes to round the bend. The radius of the curve is 150m. Compute the acceleration at the train

Solution

The speed must be converted from km/h to m/s as follows:

$$50 \text{ km/hr} = \left(50\,\frac{km}{h}\right)\left(10^3\,\frac{m}{km}\right)\left(\frac{1m}{3600}\right) = 13.89 \text{ m/s}$$

$$90 \text{ km/hr} = \left(90\,\frac{km}{h}\right)\left(10^3\,\frac{m}{km}\right)\left(\frac{1h}{3600}\right) = 25 \text{ m/s}$$

when v =13.89m/s

$$a_{\perp} = \frac{v^2}{r} = \frac{13.89^2}{150} = 1.28 \text{ m/s}^2$$

tangent acceleration $a_t = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{13.89-25}{15} = -0.47 \text{ m/s}^2$

$$\boldsymbol{a} = \sqrt{a_{\perp}^2 + a_t^2} = \sqrt{(1.28)^2 + (-0.47)^2} = 1.34 \text{ m/s}^2$$

Q) A magnetic field is applied perpendicular to an electron with a speed of 4 x10 ^{5} m/s, causing it to rotate in a circular path of radius 3m. Find the centripetal acceleration

Solution:

$$a_{\perp} = \frac{v^2}{r} = \frac{4 \text{ x}10^{\ 5}}{3} = 5.33\text{x}10^{\ 10} \text{ m/s}^2$$