Exercises(1-27):

- Check that the following chains represent composition chains for the indicated group.

a. For $(Z_{36}, +_{36})$, the group of integers modulo 36:

$$Z_{36} \supset \langle 3 \rangle \supset \langle 9 \rangle \supset \langle 18 \rangle \supset \{0\}.$$

b. For $(G_s, *)$, the group of symmetries of the square:

$$G \supset \{R_{180}, R_{360}, D_1, D_2\} \supset \{R_{360}, D_1\} \supset \{R_{360}\}.$$

c. For $(\langle a \rangle, *)$, a cyclic group of order 30:

$$\langle a \rangle \supset \langle a^5 \rangle \supset \langle a^{10} \rangle \supset \{e\}.$$

d. For $(S_3, \circ)$, the symmetric group on 3 symbols:

$$S_3 \supset \{i, (123), (132)\} \supset \{i\}.$$

- Find a composition chain for the symmetric group $(S_4, \circ)$.
- Prove that the cyclic subgroup $(\langle n \rangle, +)$ is a maximal normal subgroup of $(Z, +)$ if and only if n is a prime number.
- Establish that the following two composition chains for $(Z_{36}, +_{36})$ are equivalent:

$$Z_{24} \supset \langle 3 \rangle \supset \langle 6 \rangle \supset \langle 12 \rangle \supset \{0\},$$

$$Z_{24} \supset \langle 2 \rangle \supset \langle 4 \rangle \supset \langle 12 \rangle \supset \{0\}.$$

- Find all composition chains for $(Z_{36}, +_{36})$.
- Find all composition chains for $(G_s, *)$.