Cloud Physics Lab

LAB 9: Growth by Collision-Coalescence II

Droplet Altitudes and Trajectories

Introduction:

Droplet size and updraft strength determine the droplets fall speed relative to the ground. The ground relative fall speed of the drop is the algebraic sum of the drop's terminal velocity and the updraft velocity. Downward is negative and upward is positive. Since updraft velocity is always positive then the relative fall speed depends on the direction and magnitude of the drop's terminal speed.

In this lab, student will investigate droplet altitude and trajectories inside the cloud during the growth process.

Objective:

a) Plot and study altitude that cloud droplet can reach inside the cloud during the collision coalescence process.
b) Plot and study trajectory of cloud droplet inside the cloud.

Theory:

Drop Radius Versus Altitude

From the previous Lab, the growth equation in terms of cloud drop given by:

$$\frac{dR}{dt} = \frac{EM}{4\rho_L} u(R) \tag{1}$$

Using chain rule

$$\frac{dR}{dz} = \frac{dR}{dt}\frac{dt}{dz} \tag{2}$$

dz/dt is the vertical velocity of the drop which is its terminal velocity subtracted from the updraft speed, $U - u(R)$. Therefore

$$\frac{dR}{dz} = \frac{dR}{dt}\frac{1}{U - u(R)} \tag{3}$$

Substituting eq. (1) into eq. (3) gives

$$\frac{dR}{dz} = \frac{EM}{4\rho_l}\frac{u(R)}{U - u(R)} \tag{4}$$

Allowing $u(R) = bR$ and integrating eq. (4) gives an equation for droplet altitude versus radius,

$$z = z_o + \frac{4U\rho_l}{bEM}\left[\ln\left(\frac{R}{R_o}\right) - \frac{b}{U}\left(R - R_o\right)\right] \quad (5)$$

where R_o is the initial droplet radius, and z_o is the altitude at cloud base.

Droplet Trajectories

Droplet trajectory is how the altitude of the cloud drop changes with time. To drive such expression, we need an expression for the terminal velocity as a function of time, *u(t)*. We can derive this by the chain rule:

$$\frac{du}{dt} = \frac{du}{dR}\frac{dR}{dt} \quad (6)$$

Assuming the terminal velocity is given by $u = bR$, and using eq. (1) for dR/dt, eq. (6) becomes

$$\frac{du}{dt} = \frac{bEM}{4\rho_l}u \quad (7)$$

Integrating eq. (7) gives

$$u(t) = bR_o \exp\left(\frac{bEM}{4\rho_l}t\right) \quad (8)$$

where $bR_o = u_0$ is the terminal velocity at $t = 0$.

Recall that $dz / dt = U - u(t)$ and substitute into eq. (8) gives:

$$\frac{dz}{dt} = U - bR_o \exp\left(\frac{bEM}{4\rho_l}t\right) \quad (9)$$

Integrating again with respect to t results in

$$z = z_o + Ut + \frac{4\rho_l R_o}{EM}\left[1 - \exp\left(\frac{bEM}{4\rho_l}t\right)\right] \quad (10)$$

where z_o is the altitude at cloud base.

Materials and Procedures:

1. Run the Matlab script **Lab9a.m** to plot the height of cloud drops for different updrafts.
2. Run the Matlab script **Lab9bm** to plot the trajectory of cloud drop for different updrafts.

Analysis and Conclusions:

1. Use figure 1 to describe the behavior of cloud drop growth with height. How updraft can affect this growth?
2. Use figure 2 to describe the trajectory of a cloud drop inside the cloud. How updraft can affect the drop trajectory?

Questions:

1. What did you learn about the height and trajectory of cloud drops during the collision and coalescence process?
2. In fig. 1, explain why the growth rate is slow when the drop starts to grow until it reaches its maximum height and then start to grow faster when it starts to descend?
3. Fig. 2 indicates that the stronger the updraft, the longer the drop remains within the cloud. Stronger updrafts also lift the droplet higher into the cloud. Explain?