Material Science

Second Course

Lecture notes and problems

For

Second Grade in Department of Physics

College of Science / University of Mustansiriyah

By

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Textbook

Materials Science and Engineering: An Introduction, 8th Edition. By William D. Callister & David G. Rethwisch, 2009.

Suggested References:

Solid-State Physics: An Introduction to Principles of Materials Science 4th Edition. By Harald Ibach Hans Lüth, 2009.

Outline of this course

Introduction,

1. Diffusion,
2. **Optical properties,**
3. Electrical properties,
4. Mechanical Properties,
5. Thermal properties,
6. Problems.

❷ OPTICAL PROPERTIES

Optical property refers to a material's response to exposure to electromagnetic radiation and, in particular, to visible light. This chapter first discusses some of the basic principles and concepts relating to the nature of electromagnetic radiation and its possible interactions with solid materials. Next to be explored are the optical behaviors of metallic and nonmetallic materials in terms of their absorption, reflection, and transmission characteristics. The final sections outline luminescence, photoconductivity, and light amplification by stimulated emission of radiation (laser), the practical use of these phenomena, and optical fibers in communications.

1. Electromagnetic Radiation

In the classical sense, electromagnetic radiation is considered to be wavelike, consisting of electric and magnetic field components that are perpendicular to each other and also to the direction of propagation (Figure 2.1). Light, heat (or radiant energy), radar, radio waves, and x-rays are all forms of electromagnetic radiation. Each is characterized primarily by a specific range of wavelengths, and also according to the technique by which it is generated. The electromagnetic spectrum of radiation spans the wide range from -rays (emitted by radioactive materials) having wavelengths on the order of 10^{-12} m (10^{-3} nm), through x-rays, ultraviolet, visible, infrared, and finally radio waves with wavelengths as long as 105 m. This spectrum, on a logarithmic scale, is shown in Figure 2.2.

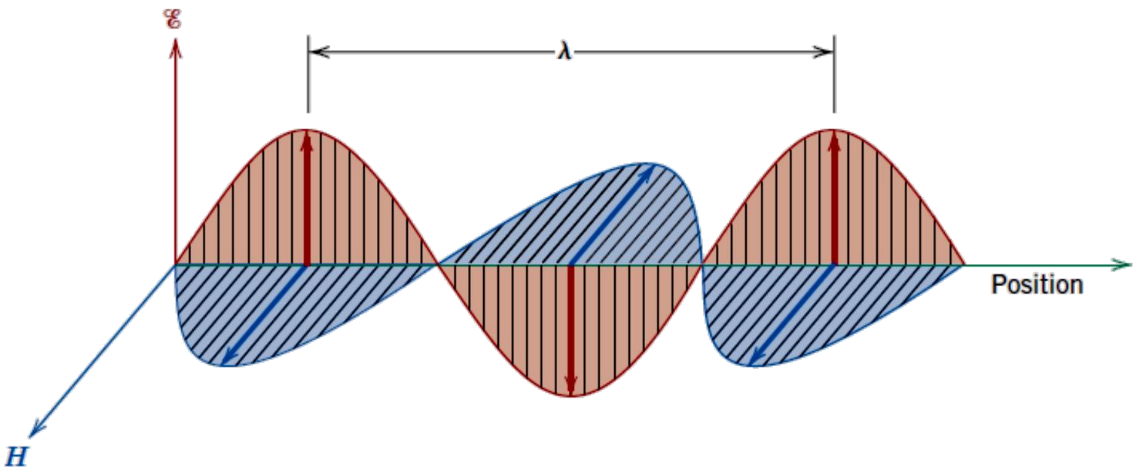

Figure 2.1. An electromagnetic wave showing electric field $\mathscr{E}$ and magnetic field **H** components, and the wavelength **λ**.

Visible light lies within a very narrow region of the spectrum, with wavelengths ranging between about 0.4 μm (4×10^{-7} m) and 0.7 μm. The perceived color is determined by wavelength; for example, radiation having a wavelength of approximately 0.4 μm appears violet, whereas green and red occur at about 0.5 and 0.65 μm, respectively. The spectral ranges for the several colors are included in Figure 2.2. White light is simply a mixture of all colors. The ensuing discussion is concerned primarily with this visible radiation, by definition the only radiation to which the eye is sensitive.

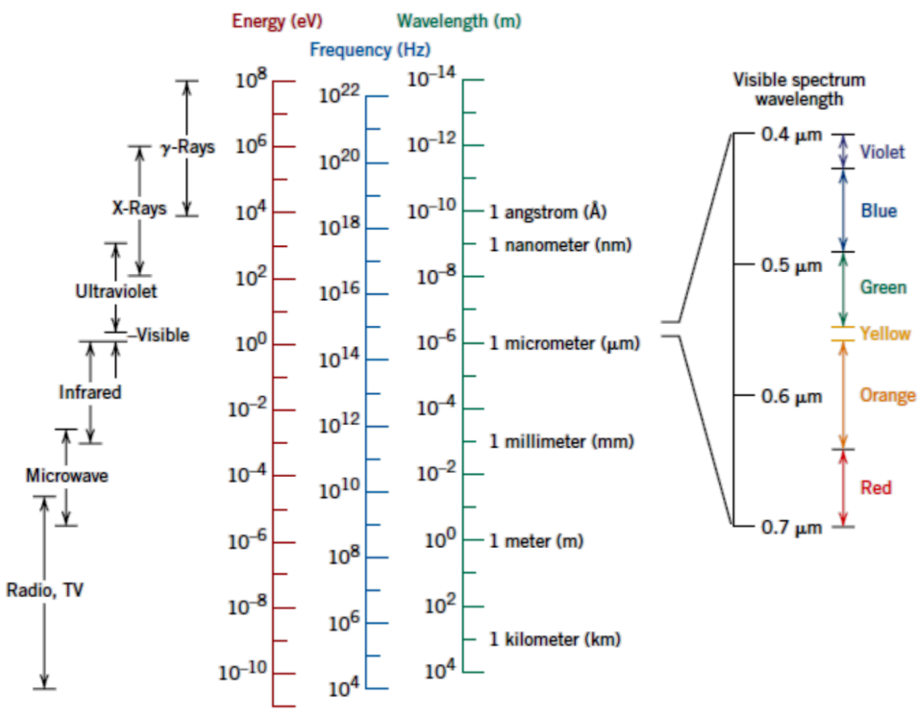

Figure 2.2. The spectrum of electromagnetic radiation, including wavelength ranges for the various colors in the visible spectrum.

All electromagnetic radiation traverses a vacuum at the same velocity, that of light—namely, 3×10^8 m/s. This velocity, c, is related to the electric permittivity of a vacuum ϵ_0 and the magnetic permeability of a vacuum μ_0 through:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \qquad 2.1$$

Thus, there is an association between the electromagnetic constant c and these electrical and magnetic constants.

Furthermore, the frequency v and the wavelength λ of the electromagnetic radiation are a function of velocity according to:

$$c = \lambda \upsilon \qquad 2.2$$

Frequency is expressed in terms of hertz (Hz), and 1 Hz = 1 cycle per second. Ranges of frequency for the various forms of electromagnetic radiation are also included in the spectrum (Figure 2.2).

Sometimes it is more convenient to view electromagnetic radiation from a quantum mechanical perspective, in which the radiation, rather than consisting of waves, is composed of groups or packets of energy, which are called photons. The energy E of a photon is said to be quantized, or can only have specific values, defined by the relationship:

$$E = h\upsilon = \frac{hc}{\lambda} \qquad 2.3$$

where h is a universal constant called **Planck's constant = 6.63×10^{-34}**. Thus, photon energy is proportional to the frequency of the radiation, or inversely proportional to the wavelength. Photon energies are also included in the electromagnetic spectrum (Figure 2.2).

When describing optical phenomena involving the interactions between radiation and matter, an explanation is often facilitated if light is treated in terms of photons. On other occasions, a wave treatment is more appropriate; at one time or another, both approaches are used in this discussion.

2. Light Interactions with Solids

When light proceeds from one medium into another (e.g., from air into a solid substance), several things happen. Some of the light radiation may be transmitted through the medium, some will be absorbed, and some will be reflected at the interface between the two media. The intensity I_0 of the beam incident to the surface of the solid medium must equal the sum of the intensities of the transmitted, absorbed, and reflected beams, denoted as I_T, I_A, and I_R, respectively, or:

$$I_0 = I_T + I_A + I_R \qquad 2.4$$

Radiation intensity, expressed in watts per square meter, corresponds to the energy being transmitted per unit of time across a unit area that is perpendicular to the direction of propagation. An alternate form of Eq. 2.4 is:

$$T + A + R = 1 \qquad 2.5$$

where T, A, and R represent, respectively, the transmissivity (I_T/I_0), absorptivity (I_A/I_0), and reflectivity (I_R/I_0), or the fractions of incident light that are transmitted, absorbed, and reflected by a material; their sum must equal unity, because all the incident light is either transmitted, absorbed, or reflected.

Materials that are capable of transmitting light with relatively little absorption and reflection are **transparent -**one can see through them. **Translucent** materials are those through which light is transmitted diffusely; that is, light is scattered within the interior, to the degree that objects are not clearly distinguishable when viewed

through a specimen of the material. Materials that are impervious to the transmission of visible light are termed **opaque.**

Bulk metals are opaque throughout the entire visible spectrum; that is, all light radiation is either absorbed or reflected. On the other hand, electrically insulating materials can be made to be transparent. Furthermore, some semiconducting materials are transparent whereas others are opaque.

3. Atomic And Electronic Interactions

The optical phenomena that occur within solid materials involve interactions between the electromagnetic radiation and atoms, ions, and/or electrons. Two of the most important of these interactions are electronic polarization and electron energy transitions.

3.1. Electronic Polarization

One component of an electromagnetic wave is simply a rapidly fluctuating electric field (Figure 2.1). For the visible range of frequencies, this electric field interacts with the electron cloud surrounding each atom within its path in such a way as to induce electronic polarization, or to shift the electron cloud relative to the nucleus of the atom with each change in direction of electric field component, as demonstrated in Figure 2.3. Two consequences of this polarization are as follows: (1) some of the radiation energy may be absorbed, and (2) light waves are retarded in velocity as they pass through the medium. The second consequence is manifested as refraction, a phenomenon to be discussed in Section 5.

Figure 2.3. Electronic polarization that results from the distortion of an atomic electron cloud by an electric field.

3.2. Electron Transitions

The absorption and emission of electromagnetic radiation may involve electron transitions from one energy state to another. For the sake of this discussion, consider an isolated atom, the electron energy diagram for which is represented in Figure 2.4. An electron may be excited from an occupied state at energy E_2 to a vacant and higher lying one, denoted E_4, by the absorption of a photon of energy. The change in energy experienced by the electron, ΔE, depends on the radiation frequency as follows:

$$\Delta E = h\nu \qquad 2.6$$

where, again, h is Planck's constant. At this point it is important that several concepts be understood. First, because the energy states for the atom are discrete, only specific ΔEs exist between the energy levels; thus, only photons of frequencies corresponding to the possible ΔEs for the atom can be absorbed by electron transitions. Furthermore, all of a photon's energy is absorbed in each excitation event.

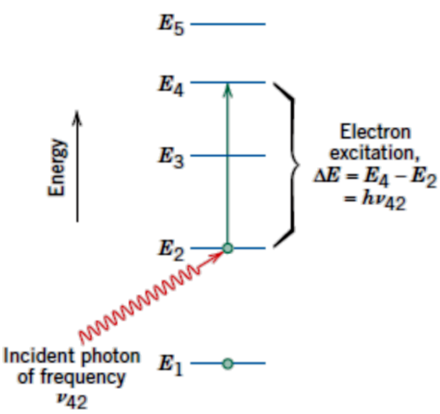

Figure 2.4. Electronic polarization that results from the distortion of an atomic electron cloud by an electric field.

A second important concept is that a stimulated electron cannot remain in an **excited state** indefinitely; after a short time, it falls or decays back into its **ground state**, or unexcited level, with a reemission of electromagnetic radiation. Several decay paths are possible, and these are discussed later. In any case, there must be a conservation of energy for absorption and emission electron transitions.

4. Optical Properties of Metals

Consider the electron energy band schemes for metals as illustrated in Figures 2.5a, b; in both cases a high-energy band is only partially filled with electrons. Metals are opaque because the incident radiation having frequencies within the visible range excites electrons into unoccupied energy states above the Fermi energy, as demonstrated in Figure 2.6a; as a consequence, the incident radiation is absorbed, in accordance with Eq. 2.6. Total absorption is within a very thin outer layer, usually less than 0.1 µm; thus, only metallic films thinner than 0.1 µm are capable of transmitting visible light.

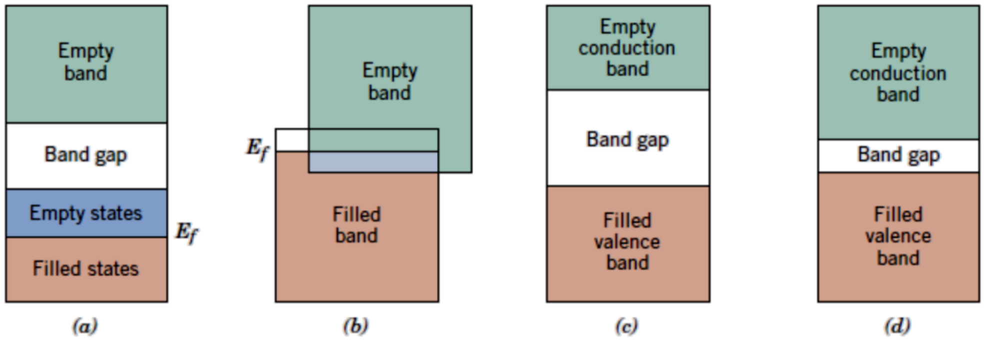

Figure 2.5. The various possible electron band structures in solids at 0 K. (*a*) The electron band structure found in metals such as copper, in which there are available electron states above and adjacent to filled states, in the same band. (*b*) The electron band structure of metals such as magnesium, wherein there is an overlap of filled and empty outer bands. (*c*) The electron band structure characteristic of insulators; the filled valence band is separated from the empty conduction band by a relatively large band gap (>2 eV). (*d*) The electron band structure found in the semiconductors, which is the same as for insulators except that the band gap is relatively narrow (<2 eV).

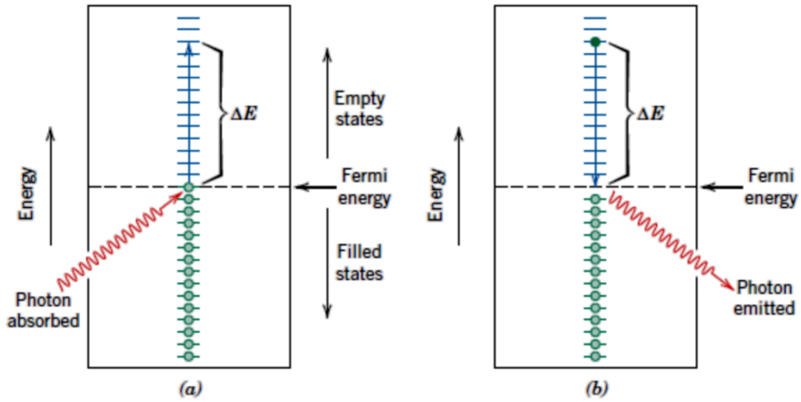

Figure 2.6. (a) Schematic representation of the mechanism of photon absorption for metallic materials in which an electron is excited into a higher-energy unoccupied state. The change in energy of the electron ΔE is equal to the energy of the photon. (b) Reemission of a photon of light by the direct transition of an electron from a high to a low energy state.

All frequencies of visible light are absorbed by metals because of the continuously available empty electron states, which permit electron transitions as in Figure 2.6a. In fact, metals are opaque to all electromagnetic radiation on the low end of the frequency spectrum, from radio waves, through infrared and the visible, and into about the middle of the ultraviolet radiation. Metals are transparent to high frequency (x- and γ-ray) radiation.

Most of the absorbed radiation is reemitted from the surface in the form of visible light of the same wavelength, which appears as reflected light; an electron transition accompanying reradiation is shown in Figure 2.6b. The reflectivity for most metals is between 0.90 and 0.95; some small fraction of the energy from electron decay processes is dissipated as heat.

Because metals are opaque and highly reflective, the perceived color is determined by the wavelength distribution of the radiation that is reflected and not absorbed. A bright silvery appearance when exposed to white light indicates that the metal is highly reflective over the entire range of the visible spectrum. In other words, for the reflected beam, the composition of these reemitted photons, in terms of frequency and number, is approximately the same as for the incident beam. Aluminum and silver are two metals that exhibit this reflective behavior. Copper and gold appear red-orange and yellow, respectively, because some of the energy associated with light photons having short wavelengths is not reemitted as visible light.

5. Optical Properties of Nonmetals

By virtue of their electron energy band structures, nonmetallic materials may be transparent to visible light. Therefore, in addition to reflection and absorption, refraction and transmission phenomena also need to be considered.

5.1. Refraction

Light that is transmitted into the interior of transparent materials experiences a decrease in velocity, and, as a result, is bent at the interface; this phenomenon is termed **refraction**. The **index of refraction** *n* of a material is defined as the ratio of the velocity in a vacuum c to the velocity in the medium υ or:

$$n = \frac{c}{\upsilon} \qquad 2.7$$

The magnitude of n (or the degree of bending) will depend on the wavelength of the light. This effect is graphically demonstrated by the familiar dispersion or separation of a beam of white light into its component colors by a glass prism. Each color is deflected by a different amount as it passes into and out of the glass, which results in the separation of the colors. Not only does the index of refraction affect the optical path of light, but also, as explained shortly, it influences the fraction of incident light that is reflected at the surface.

Just as Equation 2.1 defines the magnitude of c, an equivalent expression gives the velocity of light in a medium as:

$$\upsilon = \frac{1}{\sqrt{\epsilon\mu}} \quad 2.8$$

where ϵ and μ are, respectively, the permittivity and permeability of the particular substance. From Equation 2.7, we have

$$n = \frac{c}{\upsilon} = \frac{\sqrt{\epsilon_0\mu_0}}{\sqrt{\epsilon\mu}} = \sqrt{\epsilon_r\mu_r} \quad 2.9$$

where ϵ_r and μ_r are the dielectric constant and the relative magnetic permeability, respectively. Because most substances are only slightly magnetic, $\mu_r \approx 1$, and

$$n \cong \sqrt{\epsilon_r} \quad 2.10$$

Thus, for transparent materials, there is a relation between the index of refraction and the dielectric constant. As already mentioned, the phenomenon of refraction is related to electronic polarization (Section 3) at the relatively high frequencies for visible light; thus, the electronic component of the dielectric constant may be determined from index of refraction measurements using Eq. 2.10.

Because the retardation of electromagnetic radiation in a medium result from electronic polarization, the size of the constituent atoms or ions has a considerable influence on the magnitude of this effect -generally, the larger an atom or ion, the greater the electronic polarization, the slower the velocity, and the greater the index of refraction. The index of refraction for a typical soda–lime glass is approximately 1.5. Additions of large barium and lead ions (as BaO and PbO) to a glass will increase n significantly. For example, highly leaded glasses containing 90 wt% PbO have an index of refraction of approximately 2.1.

For crystalline ceramics that have cubic crystal structures, and for glasses, the index of refraction is independent of crystallographic direction (i.e., it is isotropic). Noncubic crystals, on the other hand, have an anisotropic n; that is, the index is greatest along the directions that have the highest density of ions. Table 21.1 gives refractive indices for several glasses, transparent ceramics, and polymers. Average values are provided for the crystalline ceramics in which *n* is anisotropic.

Table 2.1. Refractive Indices for Some Transparent Materials.

Material	Average Index of Refraction	Material	Average Index of Refraction
Ceramics		Polymers	
Silica glass	1.458	Polytetrafluoroethylene	1.35
Borosilicate (Pyrex) glass	1.47	Poly(methyl methacrylate)	1.49
Soda–lime glass	1.51	Polypropylene	1.49
Quartz (SiO_2)	1.55	Polyethylene	1.51
Dense optical flint glass	1.65	Polystyrene	1.60
Spinel ($MgAl_2O_4$)	1.72		
Periclase (MgO)	1.74		
Corundum (Al_2O_3)	1.76		

5.2. Reflection

When light radiation passes from one medium into another having a different index of refraction, some of the light is scattered at the interface between the two media even if both are transparent. The reflectivity R represents the fraction of the incident light that is reflected at the interface, or:

$$R = \frac{I_r}{I_0} \qquad 2.11$$

where I_0 and I_R are the intensities of the incident and reflected beams, respectively. If the light is normal (or perpendicular) to the interface, then:

$$R = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 \qquad 2.12$$

where n_1 and n_2 are the indices of refraction of the two media. If the incident light is not normal to the interface, R will depend on the angle of incidence. When light is transmitted from a vacuum or air into a solid *s*, then:

$$R = \left(\frac{n_s - 1}{n_s + 1}\right)^2 \qquad 2.13$$

because the index of refraction of air is very nearly unity. Thus, the higher the index of refraction of the solid, the greater the reflectivity. For typical silicate glasses, the reflectivity is approximately 0.05. Just as the index of refraction of a solid depends on the wavelength of the incident light, so also does the reflectivity vary with wavelength. Reflection losses for lenses and other optical instruments are minimized significantly by coating the reflecting surface with very thin layers of dielectric materials such as magnesium fluoride (MgF_2).

5.3. Absorption

Nonmetallic materials may be opaque or transparent to visible light; if transparent, they often appear colored. In principle, light radiation is absorbed in this group of materials by two basic mechanisms, which also influence the transmission characteristics of these nonmetals. One of these is electronic polarization (Section 3). Absorption by electronic polarization is important only at light frequencies in the vicinity of the relaxation frequency of the constituent atoms. The other mechanism involves valence band conduction band electron transitions, which depend on the electron energy band structure of the material; band structures for semiconductors and insulators were discussed later.

Absorption of a photon of light may occur by the promotion or excitation of an electron from the nearly filled valence band, across the band gap, and into an empty state within the conduction band, as demonstrated in Figure 2.7a; a free electron in the conduction band and a hole in the valence band are created. Again, the energy of excitation ΔE is related to the absorbed photon frequency through Eq. 2.6. These excitations with the accompanying absorption can take place only if the photon energy is greater than that of the band gap E_g -that is, if :

$$h\nu > E_g \qquad 2.14$$

or, in terms of wavelength,

$$\frac{hc}{\lambda} > E_g \qquad 2.15$$

The minimum wavelength for visible light, $\lambda_{(min)}$, is about 0.4 μm, and because c = 3×10^8 m/s and h = 4.13×10^{-15} eV.s, the maximum band gap energy $E_{g(max)}$ for which absorption of visible light is possible is just:

$$E_{g(max)} = \frac{hc}{\lambda_{(\min)}} = \frac{4.13\times10^{-15}\times3\times10^{8}}{4\times10^{-7}} = 3.1\text{ eV} \qquad 2.16$$

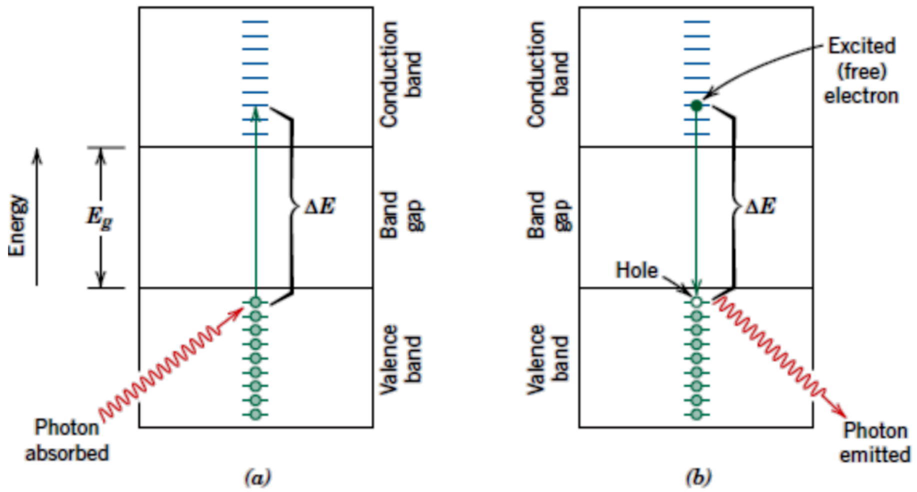

Figure 2.7. (a) Mechanism of photon absorption for nonmetallic materials in which an electron is excited across the band gap, leaving behind a hole in the valence band. The energy of the photon absorbed is ΔE, which is necessarily greater than the band gap energy E_g. (b) Emission of a photon of light by a direct electron transition across the band gap.

Or, no visible light is absorbed by nonmetallic materials having band gap energies greater than about 3.1 eV; these materials, if of high purity, will appear transparent and colorless.

On the other hand, the maximum wavelength for visible light, $\lambda_{(max)}$, is about 0.7 μm; computation of the minimum band gap energy $E_{g(min)}$ for which there is absorption of visible light is according to:

$$E_{g(min)} = \frac{hc}{\lambda_{(\max)}} = \frac{4.13\times10^{-15}\times3\times10^{8}}{7\times10^{-7}} = 1.8\text{ eV} \qquad 2.17$$

This result means that all visible light is absorbed by valence-band-to-conduction band electron transitions for semiconducting materials that have band gap energies less than about 1.8 eV; thus, these materials are opaque. Only a portion of the visible spectrum is absorbed by materials having band gap energies between 1.8 and 3.1 eV; consequently, these materials appear colored.

Every nonmetallic material becomes opaque at some wavelength, which depends on the magnitude of its E_g. For example, diamond, having a band gap of 5.6 eV, is opaque to radiation having wavelengths less than about 0.22 μm.

Interactions with light radiation can also occur in dielectric solids having wide band gaps, involving other than valence band–conduction band electron transitions. If impurities or other electrically active defects are present, electron levels within the band gap may be introduced, such as the donor and acceptor levels, except that they lie closer to the center of the band gap. Light radiation of specific wavelengths may be emitted as a result of electron transitions involving these levels within the band gap. For example, consider Figure 2.8a, which shows the valence band–conduction band electron excitation for a material that has one such impurity level. Again, the electromagnetic energy that was absorbed by this electron excitation must be dissipated in some manner; several mechanisms are possible. For one, this dissipation may occur via direct electron and hole recombination according to the reaction:

$$\text{electron} + \text{hole} \longrightarrow \text{energy}\ (\Delta E) \qquad 2.18$$

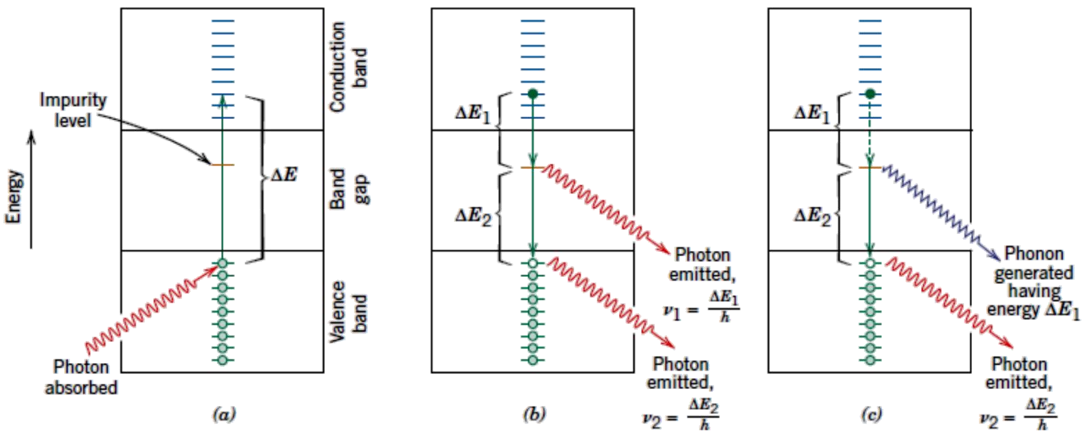

Figure 2.8. (a) Photon absorption via a valence band–conduction band electron excitation for a material that has an impurity level that lies within the band gap. (b) Emission of two photons involving electron decay first into an impurity state, and finally to the ground state. (c) Generation of both a phonon and a photon as an excited electron falls first into an impurity level and finally back to its ground state.

which is represented schematically in Figure 2.7b. In addition, multiple-step electron transitions may occur, which involve impurity levels lying within the band gap. One possibility, as indicated in Figure 2.8b, is the emission of two photons; one is emitted as the electron drops from a state in the conduction band to the impurity level, the other as it decays back into the valence band. Alternatively, one of the transitions may involve the generation of a phonon (Figure 2.8c), wherein the associated energy is dissipated in the form of heat.

The intensity of the net absorbed radiation is dependent on the character of the medium as well as the path length within. The intensity of transmitted or nonabsorbed radiation I'_T continuously decreases with distance x that the light traverses:

$$I'_T = I'_0 e^{-\beta x} \qquad 2.19$$

where I'_0 is the intensity of the nonreflected incident radiation and β, the absorption coefficient (in mm^{-1}), is characteristic of the particular material; furthermore, β varies with wavelength of the incident radiation. The distance parameter x is measured from the incident surface into the material. Materials that have large values are considered highly absorptive.

Ex (1). The fraction of nonreflected light that is transmitted through a 200 mm thickness of glass is 0.98. Calculate the absorption coefficient of this material?

Solution:

This problem calls for us to solve for β in Eq. 2.19.

$$\frac{I'_T}{I'_0} = e^{-\beta x}$$

We first of all rearrange this expression as Now taking logarithms of both sides of the preceding equation leads to:

$$\ln\left(\frac{I'_T}{I'_0}\right) = -\beta x$$

And, finally, solving for β, realizing that ($I'_T/I'_0 = 0.98\ and\ x = 200\ mm$), yield:

$$\beta = -\frac{1}{x}\ln\left(\frac{I'_T}{I'_0}\right) = -\frac{1}{200}\ln(0.98) = 1.01 \times 10^{-4}\ mm^{-1}$$

Ex (2). Zinc selenide (ZnSe) has a band gap of 2.58 eV. Over what range of wavelengths of visible light is it transparent?

Solution:

Only photons having energies of 2.58 eV or greater are absorbed by valence-band-to-conduction band electron transitions. The minimum photon energy for visible light is 1.8 eV, which corresponds to a wavelength of 0.7 µm.

$$\lambda = \frac{hc}{E} = \frac{4.13 \times 10^{-15} \times 3 \times 10^{8}}{2.58} = 4.8\ \mu m$$

Thus, pure ZnSe is transparent to visible light having wavelengths between 0.48 and 0.7 µm.

Ex (3). The fraction of nonreflected radiation that is transmitted through a 5 mm thickness of a transparent material is 0.95. If the thickness is increased to 12 mm, what fraction of light will be transmitted?

Solution:

$$\ln\left(\frac{I'_T}{I'_0}\right) = -\beta x$$

$$\beta = -\frac{1}{x}\ln\left(\frac{I'_T}{I'_0}\right) = -\frac{1}{5}\ln(0.95) = 1.03 \times 10^{-2}\ mm^{-1}$$

And computation of I'_T/I'_0 when $x = 12\ mm$:

$$\frac{I'_T}{I'_0} = e^{-\beta x} = \exp(-1.03 \times 10^{-2} \times 12) = 0.884$$

5.3.1. Absorption Mechanisms

- Absorption occurs by two mechanisms: Rayleigh scattering and Compton scattering.
- **Rayleigh scattering:** where photon interacts with the electrons orbiting an atom and is deflected without any change in photon energy. This is significant for high atomic number atoms and low photon energies. Ex.: Blue color in the sunlight gets scattered more than other colors in the visible spectrum and thus making sky look blue.
- **Tyndall effect** is where scattering occurs from particles much larger than the wavelength of light. Ex.: Clouds look white.
- **Compton scattering:** interacting photon knocks out an electron loosing some of its energy during the process. This is also significant for high atomic number atoms and low photon energies.
- **Photoelectric effect** occurs when photon energy is consumed to release an electron from atom nucleus. This effect arises from the fact that the potential energy barrier for electrons is finite at the surface of the metal. Ex.: Solar cells.

5.4. Transmission

The phenomena of absorption, reflection, and transmission may be applied to the passage of light through a transparent solid, as shown in Figure 2.9. For an incident beam of intensity I_0 that impinges on the front surface of a specimen of thickness *l* and absorption coefficient β, the transmitted intensity at the back face I_T is:

$$I_T = I_0(1 - R)^2 e^{-\beta l} \qquad 2.20$$

where R is the reflectance; for this expression, it is assumed that the same medium exists outside both front and back faces. The derivation of Eq. 2.20 is left as a homework problem.

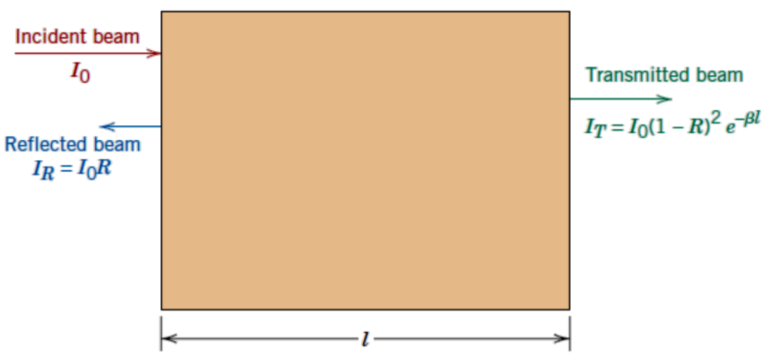

Figure 2.9. The transmission of light through a transparent medium for which there is reflection at front and back faces, as well as absorption within the medium.

Thus, the fraction of incident light that is transmitted through a transparent material depends on the losses that are incurred by absorption and reflection. Again, the sum of the reflectivity R, absorptivity A, and transmissivity T, is unity according to Eq. 2.5. Also, each of the variables R, A, and T depends on light wavelength. This is demonstrated over the visible region of the spectrum for a green glass in Figure 2.10. For example, for light having a wavelength of 0.4 μm, the fractions transmitted, absorbed, and reflected are approximately 0.90, 0.05, and 0.05, respectively. However, at 0.55 μm, the respective fractions have shifted to about 0.50, 0.48, and 0.02.

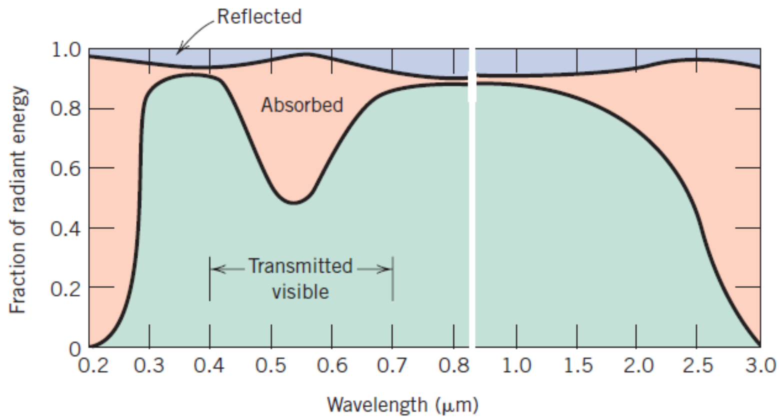

Figure 2.10. The variation with wavelength of the fractions of incident light transmitted, absorbed, and reflected through a green glass.

Ex (4). The transmissivity T of a transparent material 15 mm thick to normally incident light is 0.8. If the index of refraction of this material is 1.5, compute the thickness of material that will yield a transmissivity of 0.7. All reflection losses should be considered

Solution:

$$R = \left(\frac{n_s - 1}{n_s + 1}\right)^2 = \left(\frac{1.5 - 1}{1.5 + 1}\right)^2 = \left(\frac{n_s - 1}{n_s + 1}\right)^2 = 0.04$$

when transmissivity is $T = \frac{I_T}{I_0} = 0.8$ the absorption coefficient can be estimated as following:

$$I_T = I_0(1-R)^2 e^{-\beta l} \implies e^{-\beta l} = \frac{I_T}{I_0}\frac{1}{(1-R)^2} \implies -\beta l = \ln\left(\frac{T}{(1-R)^2}\right)$$

$$\implies \beta = -\frac{1}{l}\ln\left(\frac{I_T}{I_0}\frac{1}{(1-R)^2}\right) = -\frac{1}{15}\ln\left(\frac{0.8}{(1-0.04)^2}\right) = 9.4 \times 10^{-3}\ mm^{-1}$$

when transmissivity is $T = \frac{I_T}{I_0} = 0.7$ the thickness of material can be estimated as following:

$$I_T = I_0(1-R)^2 e^{-\beta l} \implies e^{-\beta l} = \frac{I_T}{I_0}\frac{1}{(1-R)^2} \implies -\beta l = \ln\left(\frac{T}{(1-R)^2}\right)$$

$$\implies l = -\frac{1}{\beta}\ln\left(\frac{I_T}{I_0}\frac{1}{(1-R)^2}\right) = -\frac{1}{9.4 \times 10^{-3}}\ln\left(\frac{0.7}{(1-0.04)^2}\right) = 29.3\ mm$$

5.5. Colors

Color determined by sum of frequencies of, transmitted light, and re-emitted light from electron transitions.

- Small differences in composition can lead to large differences in appearance.
- Ex: Cadmium Sulfide (CdS) has E_g= 2.4 eV, absorbs higher energy visible light (blue, violet), Red/yellow/orange is transmitted and gives it color.
- For example, high-purity single-crystal Al_2O_3(sapphire) is colourless.
- If only 0.5 - 2.0% of Cr_2O_3 add, the material looks red(ruby).
- The Cr substitutes for the Al and introduces impurity levels in the bandgap of the sapphire.
- These levels give strong absorptions at: 400 nm (green) and 600 nm (blue) leaving only red to be transmitted.

A similar technique is used to color glasses or pottery glaze by adding impurities into the molten state:

Cu^{2+}: blue-green, Cr^{3+}: green

Co^{2+}: blue-violet, Mn^{2+}: yellow

HOMEWORK

Q6: Compute the velocity of light in calcium fluoride (CaF_2), which has a dielectric constant ϵ_r and μ_r are 2.056 and 0.999 (at frequencies within the visible range), respectively.

Q7: Zinc telluride has a band gap of 2.26 eV. Over what range of wavelengths of visible light is it transparent?

Q8: The fraction of nonreflected radiation that is transmitted through a 10 mm thickness of a transparent material is 0.90. If the thickness is increased to 20 mm, what fraction of light will be transmitted?

Q9: Visible light having a wavelength of 6×10^{-7} m appears orange. Compute the frequency and energy of a photon of this light.

Q10: Briefly explain why metals are opaque to electromagnetic radiation having photon energies within the visible region of the spectrum.