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11.5 Characteristic Polynomial and Roots

❑ Polynomial comes from the Greek *poly*, "many" and medieval Latin *binomium*, "binomial".

❑ Forming a sum of several terms produces a polynomial. For example, the following is a polynomial:

$$3x^2 - 5x + 4$$

term term term
1   2   3

❑ It consists of three terms: the first is degree two, the second is degree one, and the third is degree zero.

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11.6 Polynomials in MATLAB

❑ Represented by a row vector in which the elements are the coefficients as
$$[a_n \ a_{n-1} \dots a_2 \ a_1 \ a_0]$$

❑ The a_i elements of this vector are the coefficients of the polynomial in descending order.

❑ Must include all coefficients, even if 0:

Examples:-

The polynomial

1) $s^3 - 6s^2 - 72s - 27$ is represented in MATLAB software as :

>>p = [1 -6 -72 -27]

2) 8x + 5   ,   >>p = [8 5]

3) $6x^2 - 150$, >>h = [6 0 -150]

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11.7 Roots of Polynomials

❑ We can find the roots of any polynomial with the roots(p) function where p is a row vector containing the polynomial coefficients in descending order.

Example1:

Find the roots of the polynomial
$$p_1(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$$

Solution:
The roots are found with the following two statements. We have denoted the polynomial as p1,and the roots as roots_ p1.
>> p1=[1 -10 35 -50 24] % Specify the coefficients of p1(x)
P1=
        1 -10 35   -50   24

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11.7 Roots of Polynomials

>> roots_p1=roots(p1) % Find the roots of p1(x)

roots_p1 =

        4.0000

        3.0000

        2.0000

        1.0000

We observe that MATLAB displays the polynomial coefficients as a row vector, and the roots as a column vector.

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11.7 Roots of Polynomials

Example2:

Find the roots of the polynomial

$$p_2(x) = x^5 - 7x^4 + 16x^2 + 25x + 52$$

Solution:

There is no cube term; therefore, we must enter zero as its coefficient. The roots are found with the statements below where we have defined the polynomial as p2, and the roots of this polynomial as roots_ p2.

```
>> p2=[1  -7  0  16  25  52]
P2=
        1  -7  0  16  25  52
>> roots_ p2=roots(p2)
roots_ p2 =
```

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11.7 Roots of Polynomials

```
>> roots_ p2=roots(p2)
roots_ p2 =
        6.5014
        2.7428
        -1.5711
        -0.3366 + 1.3202i
        -0.3366 - 1.3202i
```

The result indicates that this polynomial has three real roots, and two complex roots.

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11.8 Polynomial Construction from Known Roots

We can compute the coefficients of a polynomial from a given set of roots with the poly(r) function where r is a row vector containing the roots.

Example3:

It is known that the roots of a polynomial are 1,2,3 and 4. Compute the coefficients of this polynomial.

Solution:

We first define a row vector, say r3, with the given roots as elements of this vector; then, we find the coefficients with the poly(r) function as shown below.

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11.8 Polynomial Construction from Known Roots

```
>>r3=[1 2 3 4] % Specify the roots of the polynomial
r3 =
     1   2   3   4
>>poly_r3=poly(r3) % Find the polynomial coefficients
poly_r3 =
     1  -10  35  -50  24
```

We observe that these are the coefficients of the polynomial p1(x) of Example1.

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11.9 Manipulating Symbolic Expressions

1- The function **poly2sym(p)** converts the coefficient vector p to a symbolic polynomial.

>>Eq_r3=poly2sym(poly_r3) % Convert poly to expression

Eq_r3 =

 x^4 - 10*x^3 + 35*x^2 - 50*x + 24

We observe that these are the coefficients of the polynomial p1(x) of Example1.

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11.9 Manipulating Symbolic Expressions

2- The function **sym2poly(E)** converts the expression E to a polynomial coefficient vector.

>>poly_r3= sym2poly(Eq_r3) % Find the polynomial coefficients

poly_r3 =

 1   -10   35   -50   24

We observe that these are the coefficients of the polynomial p1(x) of Example1.

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11.8 Polynomial Construction from Known Roots

Example4:

It is known that the roots of a polynomial are –1, –2, –3, 4 + 5j, and 4 − 5j.Find the coefficients of this polynomial.

Solution:

We form a row vector, say r4, with the given roots, and we find the polynomial coefficients with the poly(r) function as shown below.

>> r4=[-1 -2 -3 4+5j 4-5j]

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11.8 Polynomial Construction from Known Roots

Example4:

>> r4=[-1 -2 -3 4+5j 4-5j]

r4 =

  Columns 1 through 4

 -1.0000 + 0.0000i -2.0000 + 0.0000i -3.0000 + 0.0000i   4.0000 + 5.0000i

  Column 5

  4.0000 - 5.0000i

>> poly_r4=poly(r4)

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11.8 Polynomial Construction from Known Roots

Example4:

>> poly_r4=poly(r4)

poly_r4 =

   1 -2   4 164 403 246

Therefore, the polynomial is

$$P_4(x) = x^5 - 2x^4 + 4x^3 + 164x^2 + 403x + 246$$

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11.10 Evaluation of a Polynomial at Specified Values

The polyval(p,x) function evaluates a polynomial $P(x)$ at some specified value of the independent variable x .

Evaluate the polynomial

Example5:
$$P_5(x) = x^6 - 3x^5 + 5x^3 - 4x^2 + 3x + 2$$

at x = -3 .

Solution:

>>x=sym('x');

>>y=x^6 − 3*x^5 + 5*x^3 − 4*x^2 + 3*x +2

y =

x^6 - 3*x^5 + 5*x^3 - 4*x^2 + 3*x + 2

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11.9 Evaluation of a Polynomial at Specified Values

```
>> p5=sym2poly(y)
p5 =
   1   -3   0   5   -4   3   2
>> val_minus3=polyval(p5, −3) % Evaluate p5 at x=−3.
val_minus3 =
        1280
```

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---

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11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

conv(a,b) – multiplies two polynomials a and b

[q,r]=deconv(c,d) – divides polynomial c by polynomial d and displays the quotient q and remainder r.

polyder(p) – produces the coefficients of the derivative of a polynomial p.

polyint(p) – produces the coefficients of the integral of a polynomial p.

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11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

Example6:   Let $P_1(x) = x^5 - 3x^4 + 5x^2 + 7x + 9$

$P_2(x) = 2x^6 - 8x^4 + 4x^2 + 10x + 12$

Compute the product p1.p2 with the conv(a,b) function.

Solution:

>> p1=[1 −3 0 5 7 9];

>> p2=[2 0 −8 0 4 10 12];

>> p1p2=conv(p1,p2)

p1p2 =

2 -6 -8 34 18 -24 -74 -88 78 166 174 108

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---

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

>> p1p2=conv(p1,p2)

p1p2 =

2 -6 -8 34 18 -24 -74 -88 78 166 174 108

>>y=poly2sym(p1p2)

y =

2*x^11 - 6*x^10 - 8*x^9 + 34*x^8 + 18*x^7 - 24*x^6 - 74*x^5 - 88*x^4 + 78*x^3 +

166*x^2 + 174*x + 108

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11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

<u>Example7:</u>   Let $P_3(x) = x^7 - 3x^5 + 5x^3 + 7x + 9$

$P_4(x) = 2x^6 - 8x^5 + 4x^2 + 10x + 12$

Compute the quotient p3/p4 using the deconv(p,q) function.

<u>Solution:</u>

>> p3=[1 0 −3 0 5 7 9]; p4 = [2 −8 0 0 4 10 12]; [q,r] = deconv(p3,p4)

q =

        0.5

r =

        0 4 -3 0 3 2 3

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---

11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

q =

        0.5

r =

        0 4 -3 0 3 2 3

Therefore, the quotient q(x) and remainder r(x) are :

          q(x)=0.5 $r(x) = 4x^5 - 3x^4 + 3x^2 + 2x + 3$

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11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

<u>Example8:</u>   Let $p_5 = 2x^6 - 8x^4 + 4x^2 + 10x + 12$

Compute the derivative dp_5/dx using the polyder(p) function.

<u>Solution:</u>

>> p5=[2 0 −8 0 4 10 12];

>>der_p5=polyder(p5)

der_p5 =

       12   0   -32   0   8   10

>>poly2sym(der_p5)

$$dp_5/dx = 12x^5 - 32x^3 + 8x + 10$$

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11 .Numerical analysis and curve fitting

11.10 Relations with Polynomials: conv , deconv ,polyder ,polyint

<u>Example9:</u>   Let $p_6(x) = 6x^2$

Compute the integral $\int p_6 dx$ using the polyint(p) function.

<u>Solution:</u>   OR

>>x=sym('x');   >>p6=[6 0 0]

>>y=6*x^2;   >>int_p6=polyint(p5)

>>p6=sym2poly(y)   >>poly2sym(int_p6)

p6=   int_p6 =

 6 0 0   2 0 0 0

                                  >>poly2sym(int_p6)

$$\int p_6 \, dx = 2x^3$$

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11 .Numerical analysis and curve fitting

11.11 Curve fitting

❑ Matlab also has a convenient tool for curve fitting. If we have two vectors, x and y, with paired observations, we can approximate the functional relation between them with a polynomial of some degree.

❑ If the degree is 1, the relation is linear;

❑ if it is 2, the relation is quadratic, etc.

❑ This can be done with the function polyfit().

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11 .Numerical analysis and curve fitting

❑ The following script estimates the coefficients of polynomials of order 1, 2, and 3, for a given set of observations, and plots the results in three graphs.

```
clc,clear all
x = [1 2 3 4 5 6 7 8 9]; y = [2 3 3 5 7 8 8 9 7];
x_val = linspace(0,10,100);
for degree=1:3
poly = polyfit(x,y,degree);
disp(['Coeff., case ' num2str(degree) ': ' num2str(poly)])
y_val = polyval(poly,x_val);
subplot(3,1,degree)
plot(x,y,'r*'), axis([0 10 0 10])
hold on
plot(x_val,y_val)
ylabel(['Degree: ' num2str(degree)])
end
```

11 .Numerical analysis and curve fitting

11.11 Curve fitting: poly = polyfit(x,y,degree)

<u>The output is:</u>
Coeff., case 1: 0.85   1.5278
Coeff., case 2: -0.12229   2.0729   -0.71429
Coeff., case 3: -0.053872   0.68579   -1.3318   2.8413

❑ The first two inputs to polyfit() are the vectors of X- and Y-values, and the third is the degree of the polynomial (i.e., the highest value of the exponent).
<div align="center">poly = polyfit(x,y,degree);</div>

❑ The function responds with a matrix that holds one more element than the degree.
❑ The elements of the matrix are the coefficients of the estimated polynomial.
❑ For example, in the third case above

$$y = -0.053872x^3 + 0.68579x^2 - 1.3318x + 2.8413$$

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11.11 Curve fitting: poly = polyfit(x,y,degree)

❑ The function polyval() uses a matrix of coefficients, poly above, and returns Y-values for given X-values.
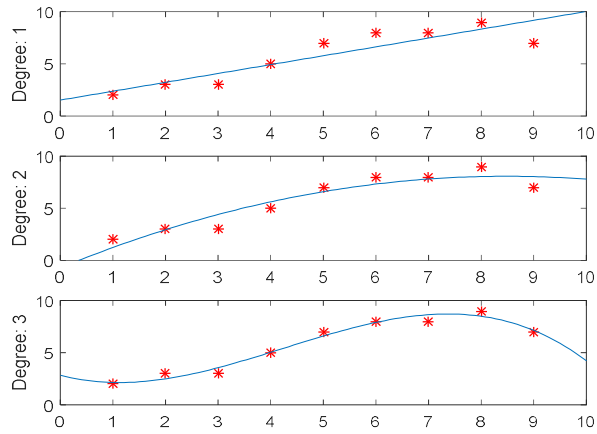<div align="center">y_val = polyval(poly,x_val);</div>

❑ Figure 11-1 shows the resulting three plots. The red markers are the same in all three cases, but the curves correspond to the fitted polynomials.

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11.11 Curve fitting: poly = polyfit(x,y,degree)

Figure 11-1

Examples of curve
fitting

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