2nd course
Lecture No.7

Rossby waves

The west-to-east jet stream can meander poleward and equatorward as Rossby waves, due to barotropic and baroclinic instability. Such waves in the upper-air (jet-stream) flow can create mid-latitude cyclones at the surface.

Rossby waves result from the interplay between inertia (trying to make the jet stream continue in the direction it was deflected) and a restoring force (acting opposite to the deflection). For Rossby waves, the restoring force can be explained by the conservation of potential vorticity, which depends on both the Coriolis parameter and the layer thickness (related to layer static stability). **Baroclinic instability** considers both restoring factors, while **barotropic instability** is a simpler approximation that considers only the Coriolis effect.

Barotropic Instability

Consider tropospheric air of constant depth Δz (≈11 km). For this situation, the conservation of potential vorticity can be written as follow:

$$\left[\frac{M}{R} + f_c\right]_{initial} = \left[\frac{M}{R} + f_c\right]_{later}$$

where jet-stream wind speed M divided by radius of curvature R gives the relative vorticity, and f_c is the Coriolis parameter (which is a function of latitude).

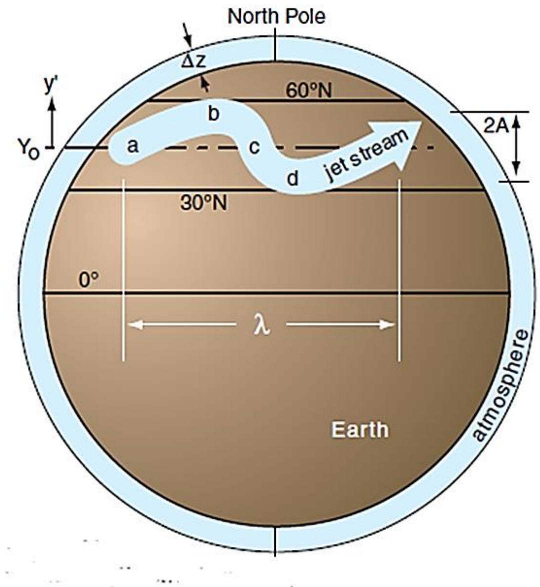

Figure 1.

(Location **a** in Fig. 1) Consider a jet stream at initial latitude *Yo* moving in a straight line from the southwest. At that latitude it has a certain value of the Coriolis parameter, but no relative vorticity ($M/R = 0$, because $R = \infty$ for a straight line). But *fc* increases as the air moves poleward, thus the M/R term on the right side of eq. (1) must become smaller than its initial value (i.e., it becomes negative) so that the sum on the right side still equals the initial value on the left side. curvature *R* gives the relative vorticity, and *fc* is the Coriolis parameter (which is a function of latitude.

(Location **b**) We interpret negative curvature as anticyclonic (clockwise turning in the N. Hemisphere). This points the jet stream equatorward.

(Location **c**) As the air approaches its starting latitude, its Coriolis parameter decreases toward its starting value. This allows the flow to become a straight line again at location c. But now the wind is blowing from the northwest, not the southwest.

(Location **d**) As the air overshoots equatorward, *fc* gets smaller, requiring a positive M/R (cyclonic curvature) to maintain constant potential vorticity. This turns the jet stream back toward its starting latitude, where the cycle repeats. The flow is said to be **barotropically unstable**, because even pure, non-meandering zonal flow, if perturbed just a little bit from its starting latitude, will respond by meandering north and south. This north-south (meridional) oscillation of the west-to-east jet stream creates the wavy flow pattern we call a **Rossby wave** or a **planetary wave**. Because the restoring force was related to the change of Coriolis parameter with latitude, it is useful to define a beta parameter as:

$$\beta = \frac{\Delta f_c}{\Delta y} = \frac{2 \cdot \Omega}{R_{earth}} \cdot \cos\phi \qquad \text{............(2)}$$

where the average radius of the Earth is REarth =6371km.

For $2 \cdot \Omega / R_{Earth} = 2.29 \times 10^{-11}\ m^{-1} \cdot s^{-1}$,

one finds that β is roughly (1.5 to 2) $\times 10^{-11}\ m^{-1} \cdot s^{-1}$.

The wave path in Fig. 1 can be approximated with a simple cosine function:

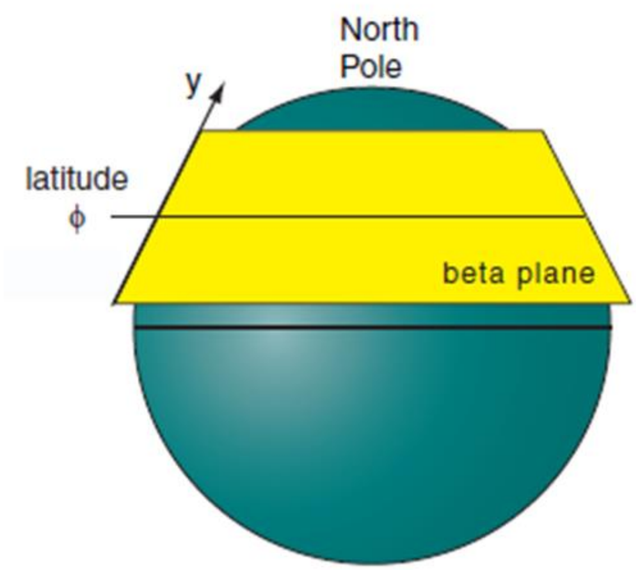

Figure 2

HIGHER MATH • The Beta Plane

Here is how you can get β using the definition of the Coriolis parameter f_c (eq. 10.16):

$$f_c = 2\,\Omega \sin\phi$$

where φ is latitude.

Since y is the distance along the perimeter of a circle of radius R_{Earth}, recall from geometry that

$$y = R_{Earth} \cdot \phi$$

for φ in radians.

Rearrange this to solve for φ, and then plug into the first equation to give:

$$f_c = 2\,\Omega \sin(y/R_{Earth})$$

By definition of β, take the derivative to find

$$\beta = \frac{\partial f_c}{\partial y} = \frac{2\cdot\Omega}{R_{earth}} \cdot \cos\left(\frac{y}{R_{earth}}\right)$$

Finally, use the second equation above to give:

$$\boxed{\beta = \frac{2\cdot\Omega}{R_{earth}} \cdot \cos\phi} \qquad \bullet(11.35)$$

For a small range of latitudes, β is nearly constant. Some theoretical derivations assume constant beta, which has the same effect as assuming that the earth is shaped like a cone. The name for this lamp-shade shaped surface is the **beta plane**.