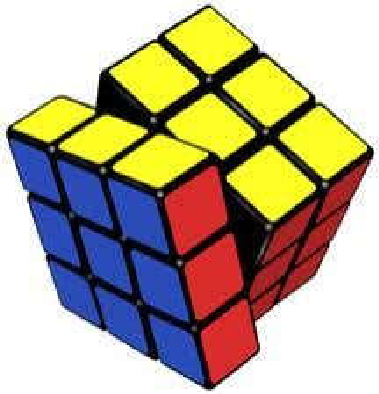
Group theory



What is group ?

- Group theory is the study of a set of elements present in a group, in Mathematics. A group's concept is fundamental to abstract algebra. Other familiar algebraic structures namely rings, fields, and vector spaces can be recognized as groups provided with additional operations and axioms.

- If G is a nonempty set, a binary operation * on G

is a function * : G x G into G

*(x,y) = x*y for all x & y є G

The above property called closure property and if its satisfied we call G is closed under *.

- A mathematic system : we call (G,*) is a mathematic system if its satisfied the closure property.
- **Examples**:
- 1- (N,+) is a mathematic system

  2- (N,-) is not mathematic system, show that?

Definition

A nonempty set G with a binary operation (*) is called a group denoted by (G,*)

If satisfied

1 – closure property

2- Associative Low

3- Existence of identity

4- Existence of inverse

Properties of Group

Closure : **For all y, x ϵ G we have x * y ϵ G**	**Associativity** (a*b)*c = a*(b*c) , for all a, b, c **ϵ** G.
An Identity element exists : e ϵ G |e * x = x * e = x	**An Inverse element exists** For all x ϵ G there exists y ϵ G s.t x * y = e ϵ G

Example : show that (Z , +) is group

- 1- for all a & b ϵ Z s.t a+b ϵ Z then Z is closed
- 2- for all a ,b , c ϵ Z s.t (a+b)+c = a+(b+c) then + is associative
- 3- 0 ϵ Z s.t a+0 = 0 + a = a for all a ϵ Z (identity element)
- 4- for all x ϵ Z there exists x^{-1}(=-x) ϵ Z s.t x+ x^{-1} = 0

Hence Z is a group

Definition (Commutative group)

A group $G,*$ is called a Commutative group iff $a*b=b*a, \forall a,b \in G$.

Examples

- $(Z,+)$, $(Q,+)$, $(C,+)$ are commutative groups .
- (N,+) is not a group. Why?

H.W

- Let $G=\{a,b,c,d\}$ be a set. Define a binary operation $*$ on G by the following

*	A	B	C	D
A	A	B	C	D
B	B	C	D	A
C	C	D	A	B
D	D	A	B	C

- Is $G,*$ a commutative group?